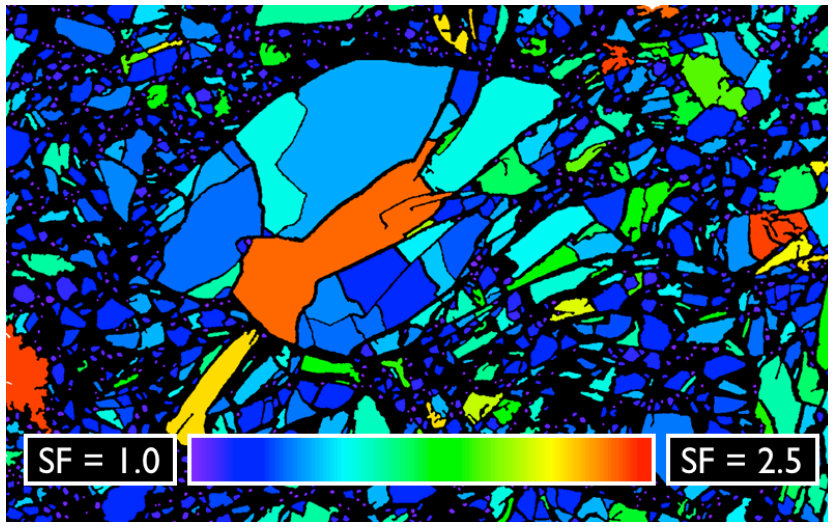
TSK19 HALLE

Workshop 2

Tuesday, March, 7, 2022



Selected topics in image analysis of deformed rocks

(2)

Shape analysis
Grain size distributions
Spatial distributions

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Schedule

Renée – Lectures

10:00-10:30 shape analysis
10:30-11:00 discussion & break
11:00-11:30 grainsize
11:30-12:00 discussion & break
12:00-12:30 phase distributions & correlations
12:30-14:00 discussion & lunch

Rüdiger – Lab

14:00-15:30 using Fiji / imageJ
15:30-16:00 break
16:00-17:00 playtime (with your own data)

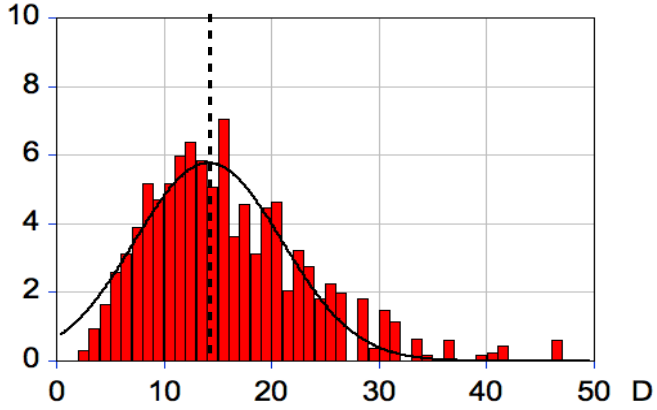
I
preparing the input

grain size is always distributed

... about a mean grain size
usually within one order of magnitude

=> 'the grain size'

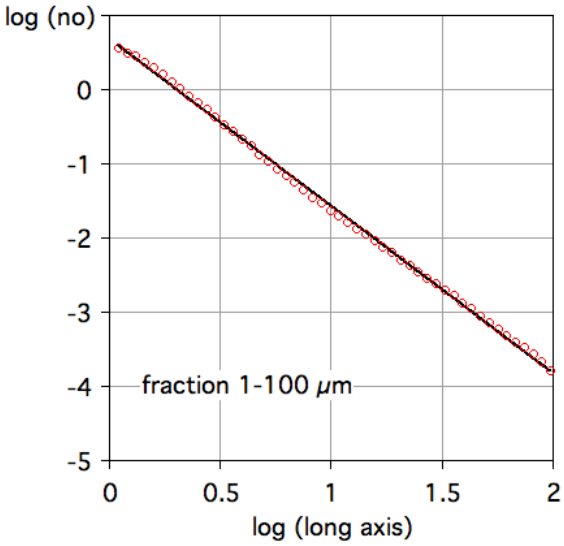
(mean, mode of histogram, 2D, 3D)



... with a constant ratio of grain size
over more two orders of magnitude

=> 'fractal dimension'

(slope of log-log plot, 2D, 3D)



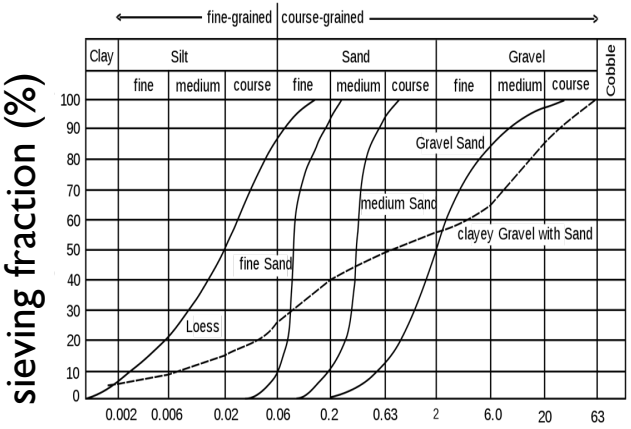
measuring grain size in 3D

using sieves, particle analyzer, tomography

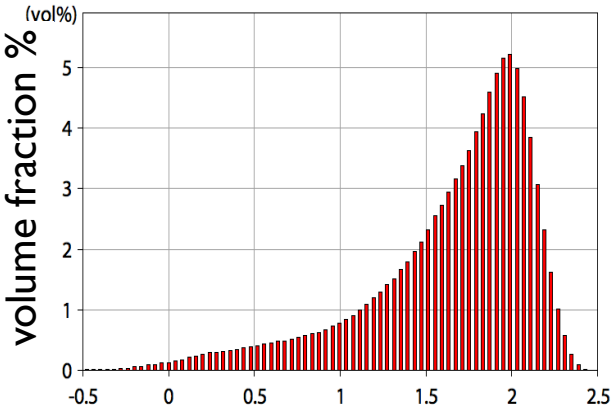
Stainless Steel Sieves

laser diffraction particle size analyser

Xray tomography nano / micro CT system



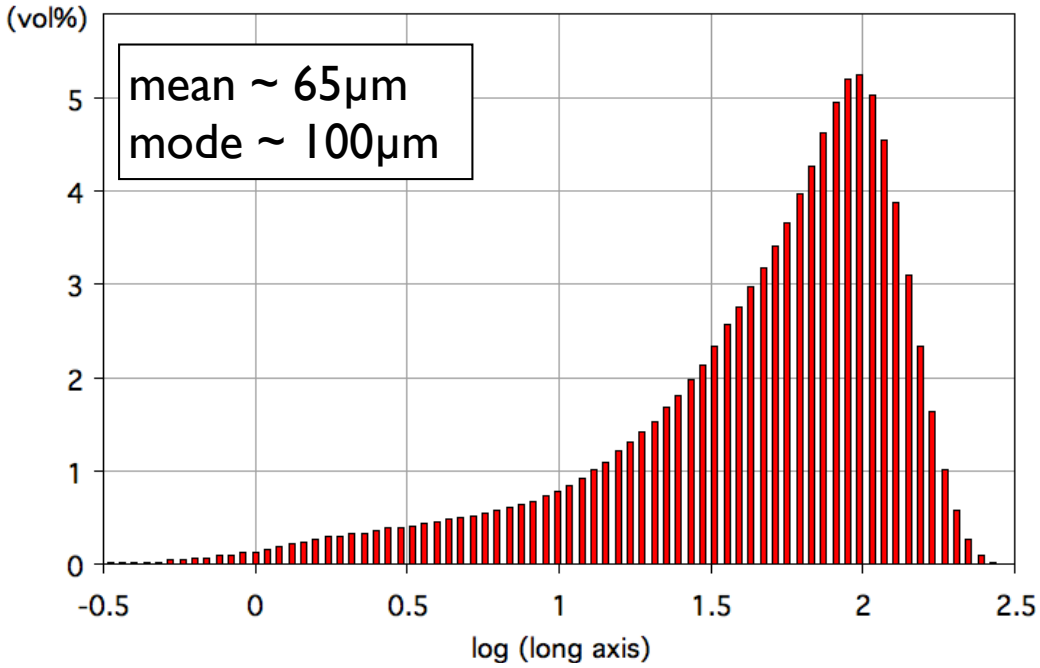
grain / sieve size (mm)

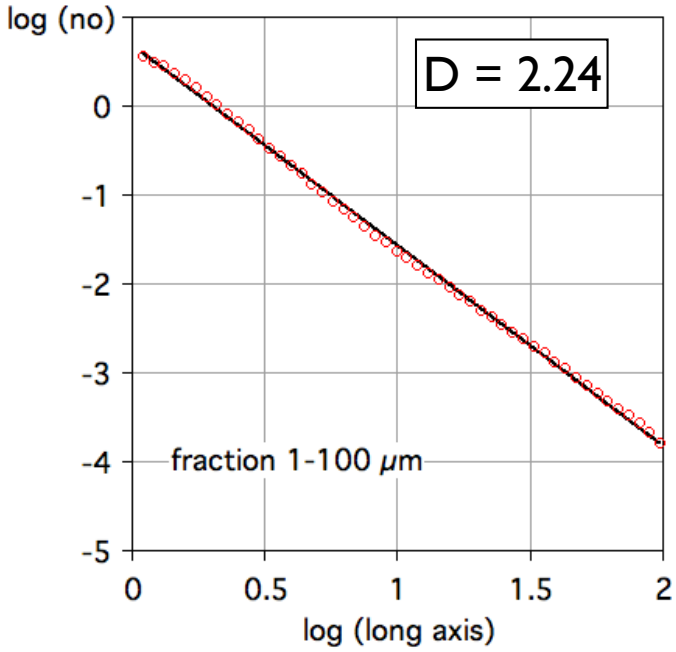


log diameter

3D visualization

fractal dimension from sieved data



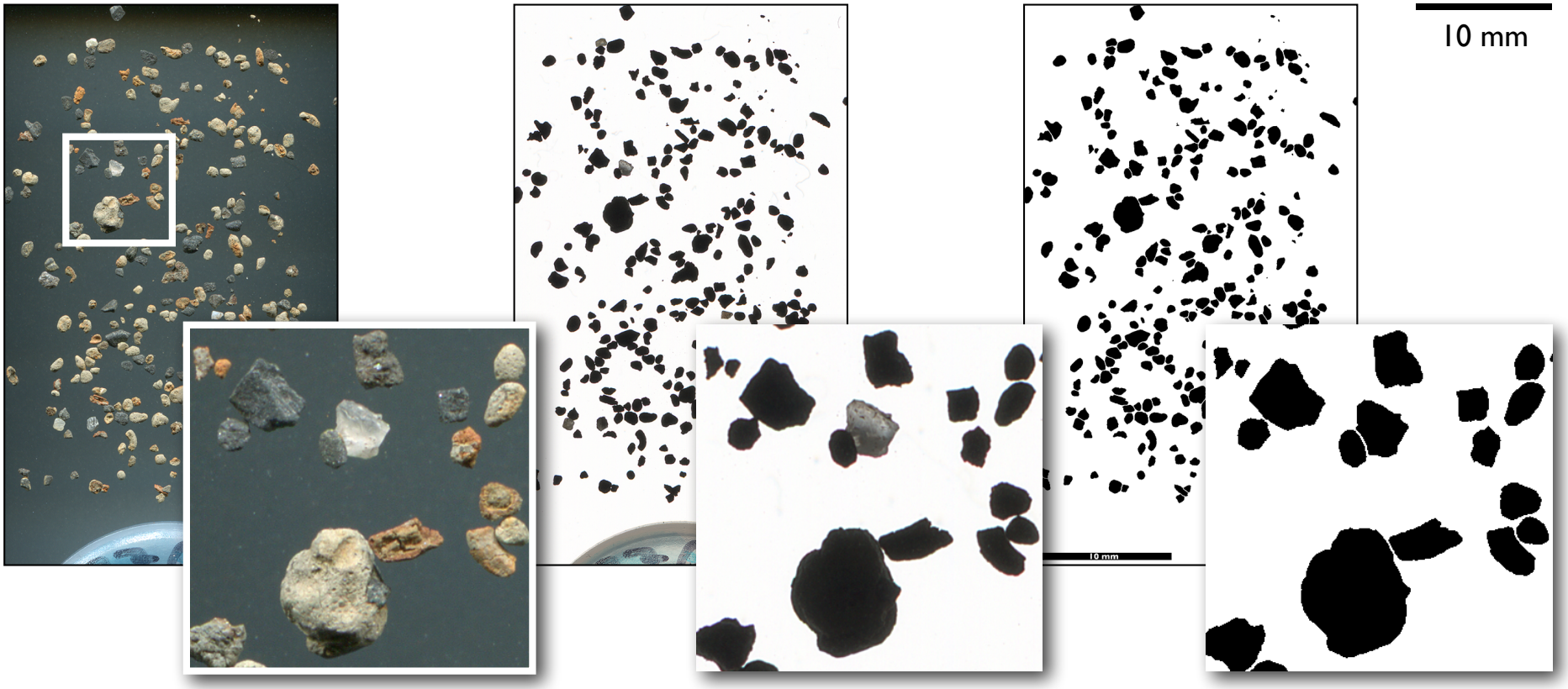


measuring grain size in 2D projection

incident light

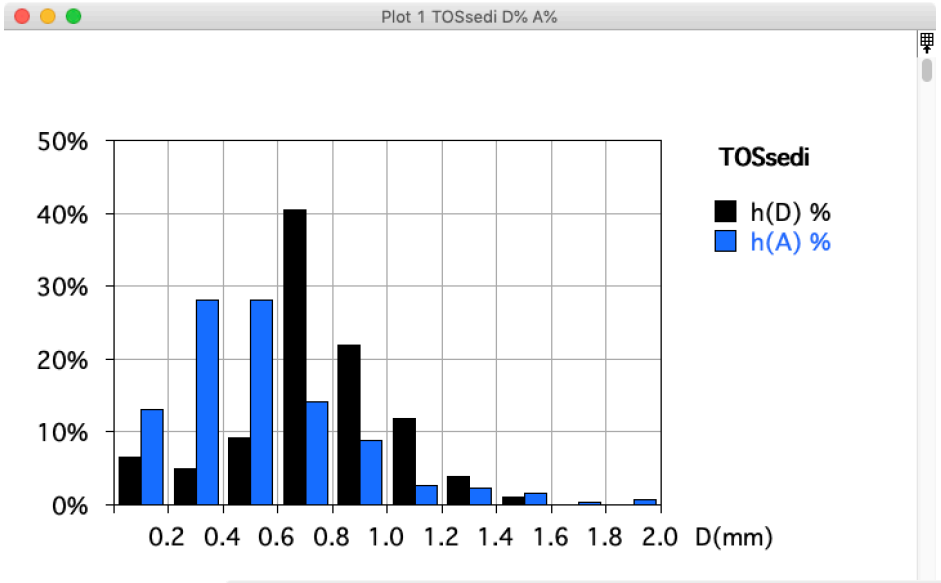
transmission

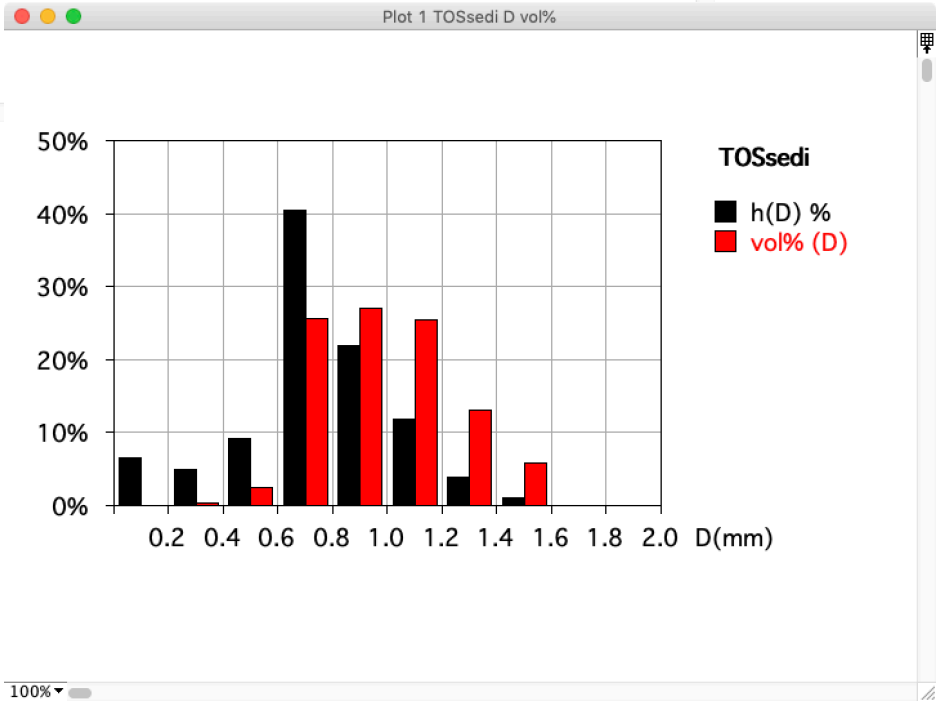
bitmap



mean grain size from projection

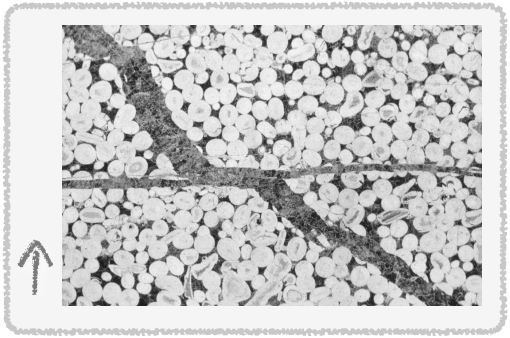
bitmap



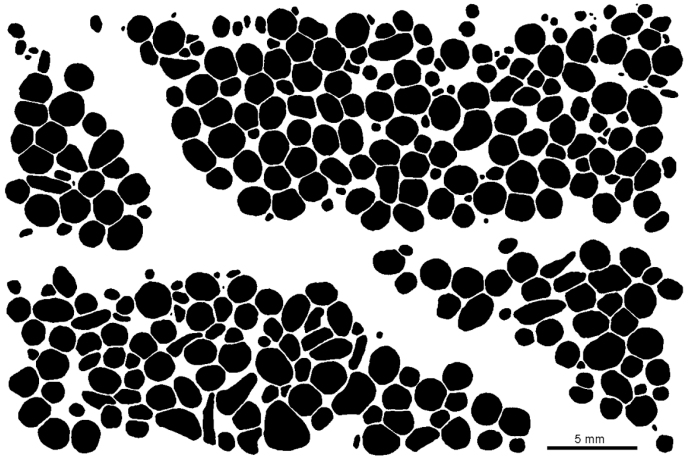


mean grain size from 2D section

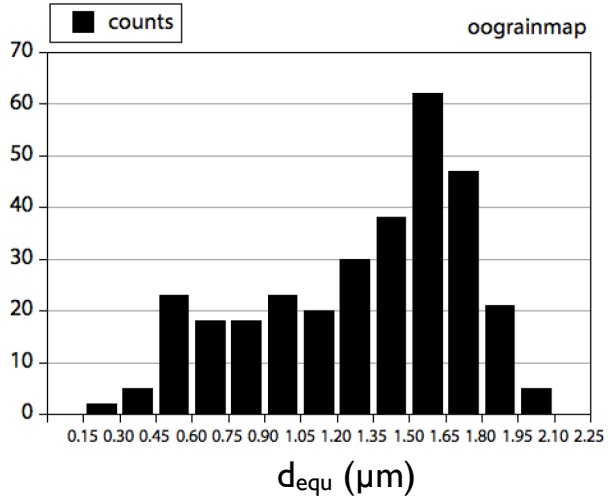
using thin sections or polished surfaces



thin section



segmentation



histogramm of diameters

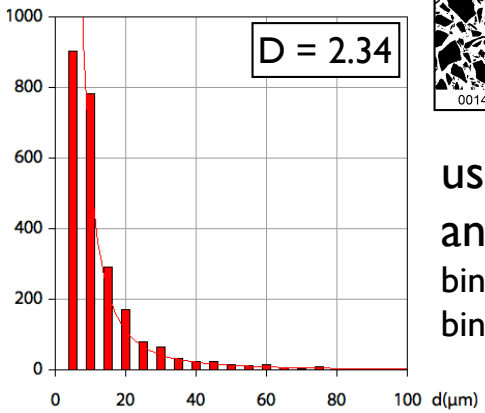
	d_{equ} (mm)
Points	313
Mean	1.3075
Median	1.4323
RMS	1.3771
Std Deviation	0.43268
Variance	0.18721
Skewness	-0.50879
Kurtosis	-0.68658

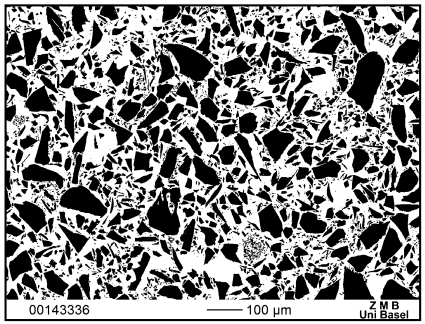
statistics

1.3075 mm

grain size !

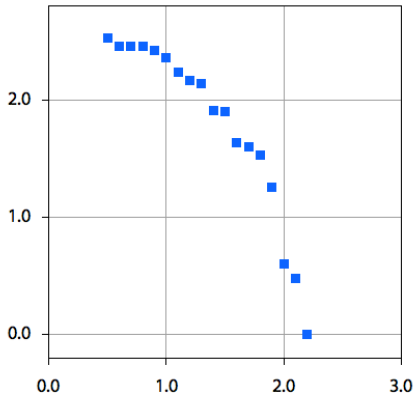
fractal dimension from 2D sections





using linear diameters
and linear numbers
binning of d_{equ} (µm)
bin width Δd = constant

(linear data - plotted on linear x- and y- axes)

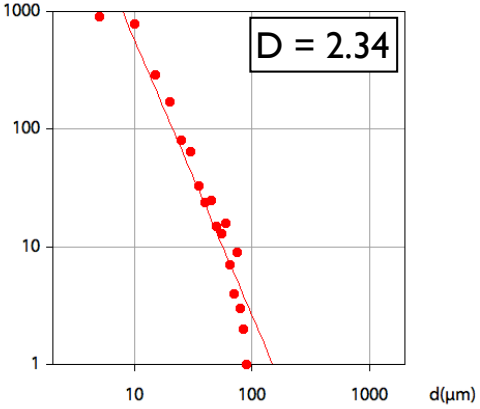


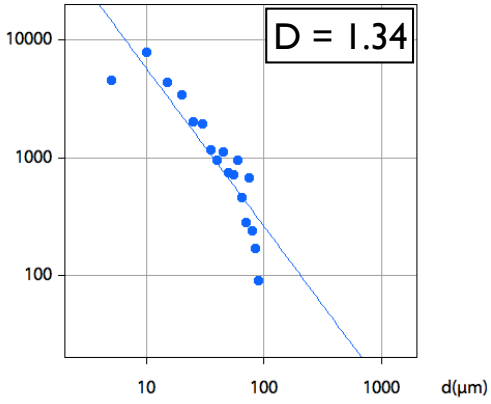
using log diameters
and log numbers
binning of log (d_{equ})
bin width $\Delta \log(d)$ = constant

(log data - plotted on linear x- and y- axes)

power law fit on linear data

n vs. $d \rightarrow D_{3d}$ $\quad (n \cdot d)$ vs. $d \rightarrow D_{2d}$

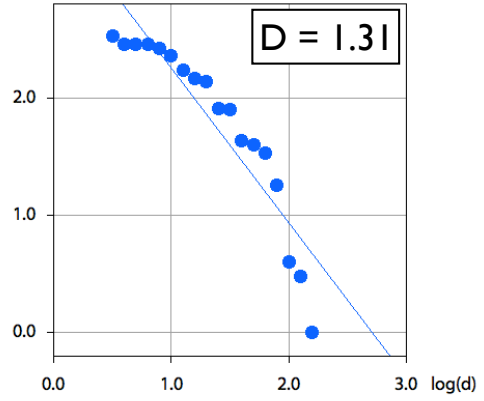




(linear data - plotted on logarithmic x- and y- axes)

linear law fit on log data

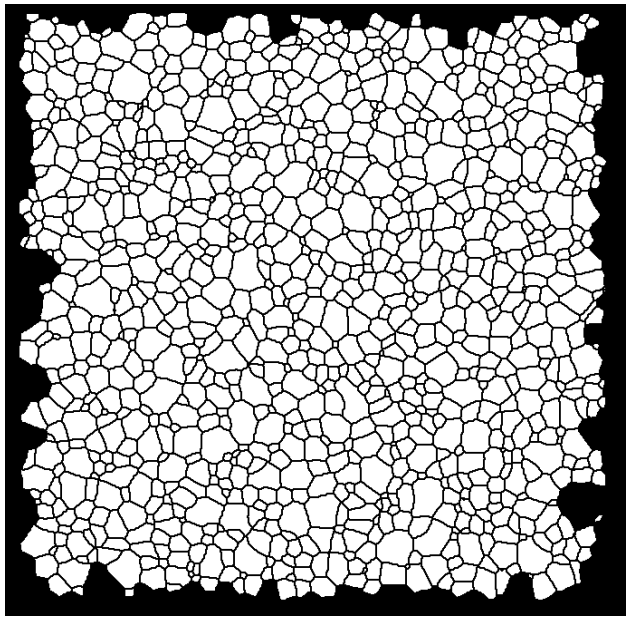
$\log(n)$ vs. $\log(d) \rightarrow D_{2d}$

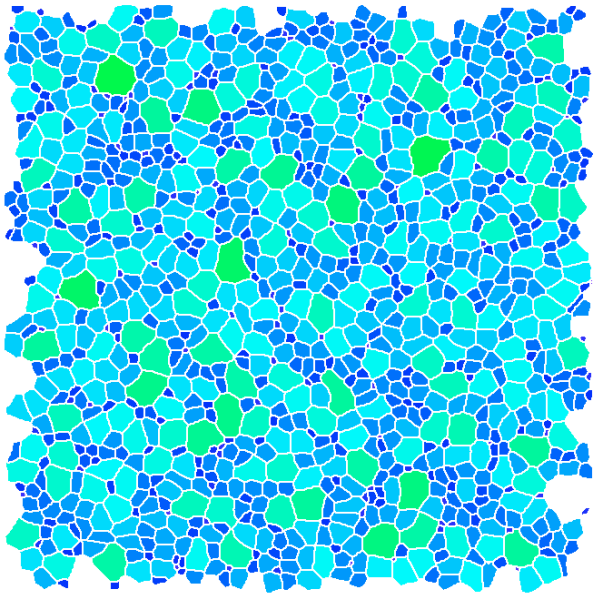


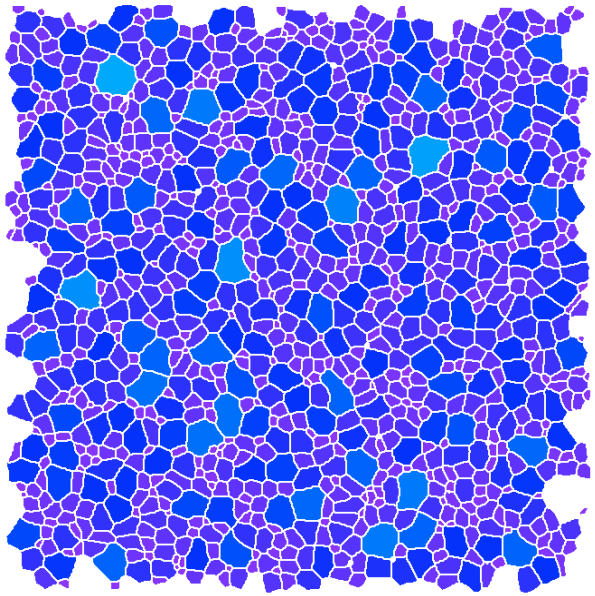
(log data - plotted on linear x- and y- axes)

2
basic grainsize

grain size is distributed







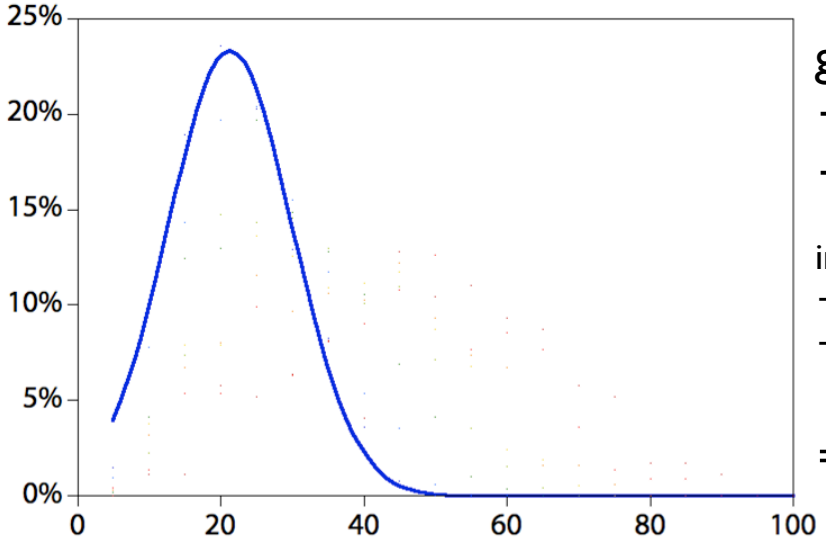
0 88 px
diameter

0 5700 sqpx
area

grain growth kinetics

Chen LQ, Yang W (1994)
Computer-simulation of the domain dynamics of a quenched system with a large number of nonconserved order parameters—the grain-growth kinetics.
Phys Rev B 50: 15752--15756

https://www.youtube.com/watch?v=p0rY2r0E_2k

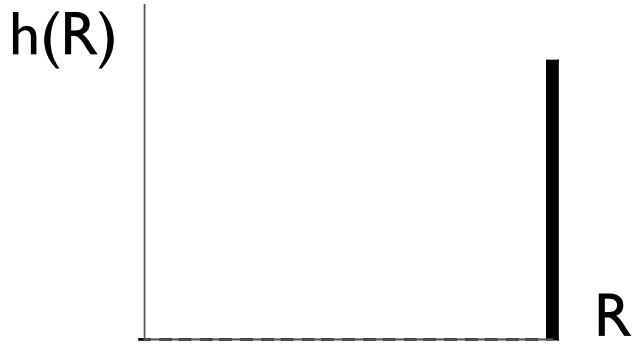


grain growth
→ increasing average size
→ increasing spread

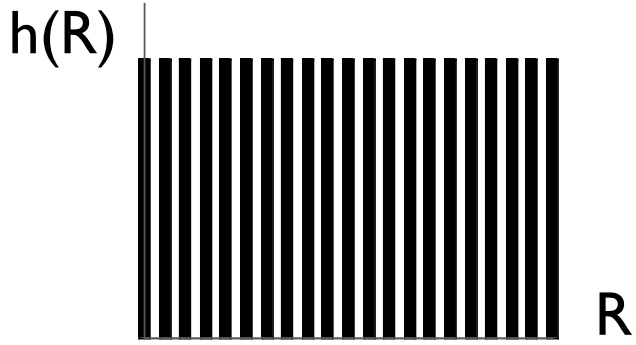
in terms of normal distribution:
→ increasing mean (μ)
→ increasing standard deviation (σ)

⇒ distribution matters

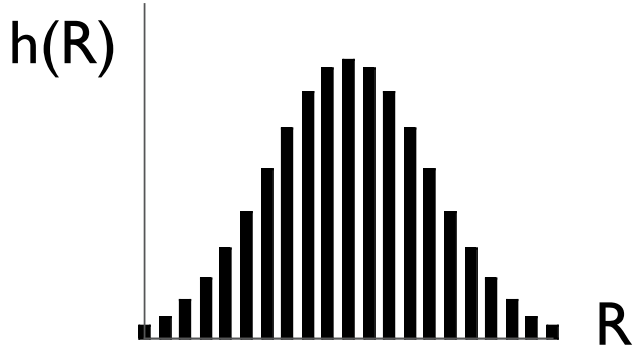
size distributions



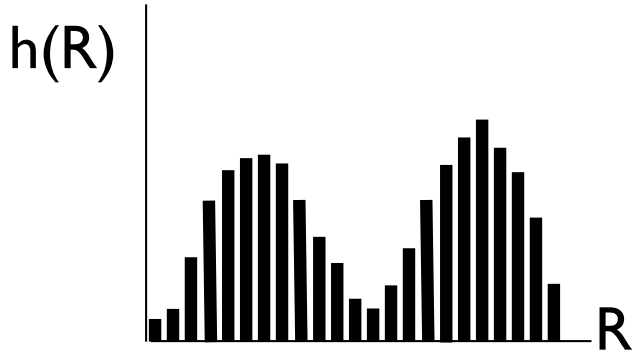
size distribution h(R): monodisperse (= Delta function)
only one grain size R
perfectly sorted grain size distribution



size distribution h(R): uniform
random distribution of size R
very poorly sorted grain size distribution



size distribution h(R): Gaussian normal
$1/\sqrt{(2\pi\sigma^2)} \cdot \exp(-((x-\mu)^2/(2\sigma^2)))$
preferred / mean grain size Rm, dispersion σR
moderately sorted grain size distribution

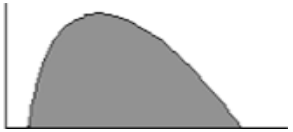


size distribution h(R): bimodal
two populations of grains
generated by mixing or two different processes

pro memoria: statistical measures

	for list of values	for grouped data
mean	Arithmetic mean $\mu = 1/n \Sigma x_i$ Geometric mean $g = \sqrt[n]{(\Pi x_i)}$ Harmonic mean $h = n \cdot (\Sigma (x_i)\text{-}1)^{-1}$	$\mu = 1/\Sigma h(x_i) \cdot \Sigma [h(x_i) \cdot x_i]$ Note: if $\Sigma h(x_i) = 1.00$ $\mu = \Sigma [h(x_i) \cdot x_i]$
median		(n = odd) $med(x) = m = x_{(n+1/2)}$ (n = even) $med(x) = m = 1/2 \cdot (x_{(n/2)} + x_{(n/2+1)})$
mode	Note: mode is not defined	mode = x of $h(x_i)max$
RMS	$RMS = (1/n \cdot \sum xi^2)^{-1}$	$RMS = (1/\Sigma h(x_i) \cdot (\sum x_i^2 - \mu)) - 1$ where $\mu = 1/\Sigma h(xi) \cdot \Sigma [h(xi) \cdot xi]$ (= Arithmetic mean)
variance	$\sigma^2 = 1/n \cdot \Sigma (xi-\mu)^2$	$\sigma^2 = 1/\Sigma h(x_i) \cdot \Sigma [h(x_i) \cdot (x_i-\mu)^2]$ if $\Sigma h(x_i) = 1.00$: $\sigma^2 = \Sigma [h(x_i) \cdot (x_i-\mu)^2]$
standard deviation	$\sigma = \sqrt{\sigma^2}$	$\sigma = \sqrt{\sigma^2}$
skewness	$sk = 1/\sigma^3 \cdot 1/n \cdot \Sigma (x_i-\mu)^3$	$sk = 1/\sigma^3 \cdot 1/\Sigma h(x_i) \cdot \Sigma [h(x_i) \cdot (x_i-\mu)^3]$ if $\Sigma h(x_i) = 1.00$ $sk = 1/\sigma^3 \cdot \Sigma [h(x_i) \cdot (x_i-\mu)^3]$
kurtosis	$k = 1/\sigma^4 \cdot 1/n \cdot \Sigma (x_i-\mu)^4$	$k = 1/\sigma^4 \cdot 1/\Sigma h(x_i) \cdot \Sigma [h(x_i) \cdot (x_i-\mu)^4]$ if $\Sigma h(x_i) = 1.00$ $k = 1/\sigma^4 \cdot \Sigma [h(x_i) \cdot (x_i-\mu)^4]$

negatively skewed (sk < 0)



positively skewed(sk > 0)

leptokurtic (k > 0)

platykurtic (k < 0)

the (in)famous 'mean grain size'

arithmetic mean	$\bar{X}$	=	$1/n \cdot \sum x_i$
geometric mean	G	=	$\sqrt[n]{\prod x_i}$
harmonic mean	H	=	$1 / (1/n \cdot \sum 1/x_i)$
		=	$n / \sum 1/x_i$
root-mean-square	RMS	=	$\sqrt{ (1/n \cdot \sum x_i^2)}$

$\sum$ = sum

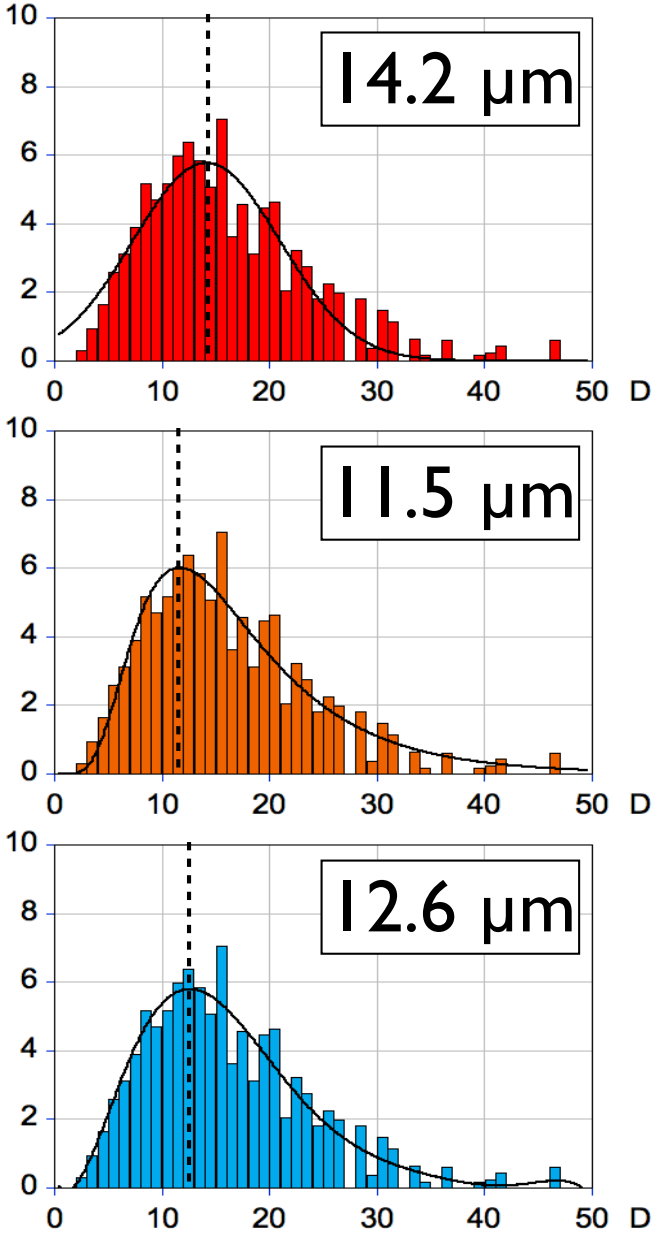
$\prod$ = product

i = 1, ... n

$$RMS > \bar{X} \geq G \geq H$$

median		=	$x_{(n+1)/2}$ if n = odd
		=	$(x_{n/2} + x_{n/2+1}) /$ if n = even
mode		=	most frequent value

finding the mode by curve fitting



Normal curve fit

$$\frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left(-\frac{(\mu - x)^2}{2\sigma^2}\right)$$

Lognormal curve fit

$$\frac{1}{x\sqrt{2\pi\sigma^2}} \cdot \exp\left(-\frac{(\ln(x)-\mu)^2}{2\sigma^2}\right)$$

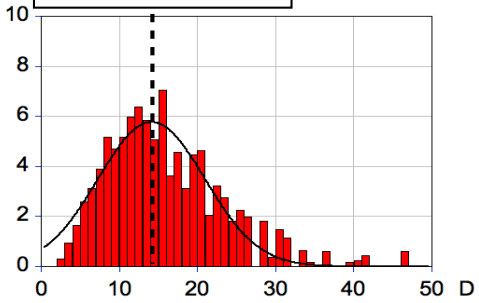
Polynomial curve fit

$$m_0 + m_1 x + m_2 x^2 + m_3 x^3 + \ldots.$$

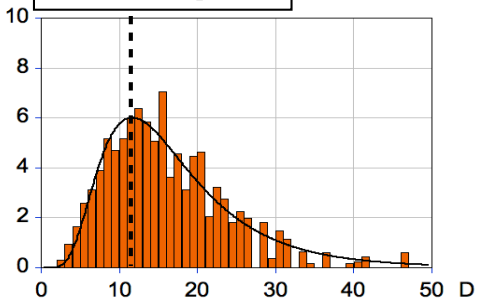
in all cases: fit through center of bin !

finding mode by non-parametric fit

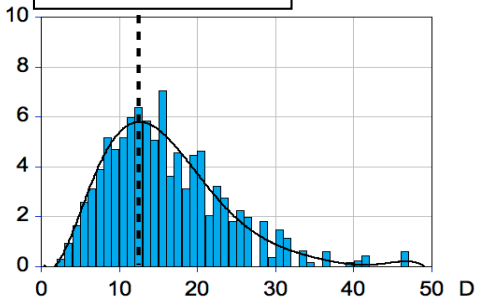
14.2 µm



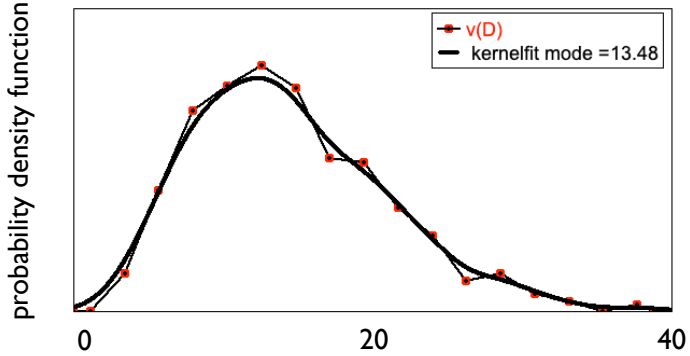
11.5 µm



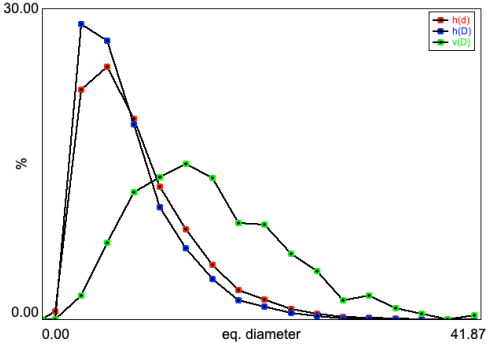
12.6 µm



13.4 µm



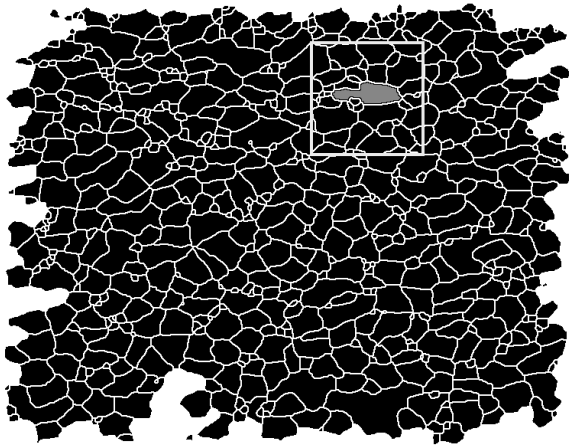
kernel sigma ≈ 1.5 µm

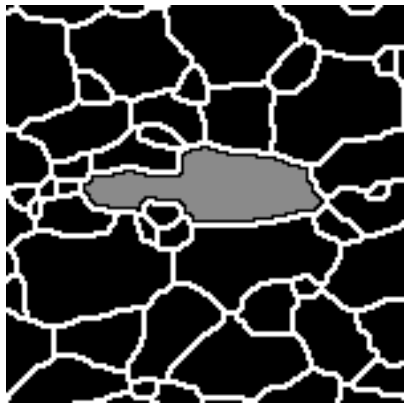


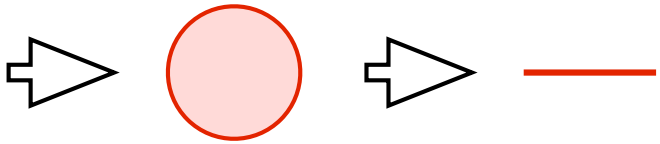
Results

	Area	Perim.	EqDia	bin	bincenter	h(d)	h(D)	v(D)	h(D)*	v(D)*
1	27.000	20.435	7.771	0.000	1.250	0.737	0.000	0.000	-12.406	-0.143
2	7.000	9.536	4.588	2.500	3.750	22.129	28.479	2.311	24.946	2.308
3	13.250	17.778	6.285	5.000	6.250	24.332	26.848	7.354	23.517	7.343
4	91.500	47.799	13.318	7.500	8.750	19.350	18.839	12.231	16.502	12.213
5	19.750	16.657	6.808	10.000	11.250	12.819	10.820	13.721	9.478	13.701
6	9.750	13.485	5.439	12.500	13.750	8.642	6.833	14.973	5.986	14.952
7	7.500	11.657	4.939	15.000	16.250	5.248	3.915	13.624	3.430	13.604
8	5.000	9.364	4.277	17.500	18.750	2.795	1.795	9.324	1.572	9.311
9	13.250	14.364	5.930	20.000	21.250	1.876	1.231	9.102	1.078	9.089
10	1.250	3.828	2.543	22.500	23.750	1.002	0.622	6.309	0.545	6.300
11	26.500	21.971	7.856	25.000	26.250	0.547	0.341	4.608	0.299	4.601
12	3.250	6.950	3.604	27.500	28.750	0.220	0.106	1.861	0.093	1.858
13	84.500	39.506	12.565	30.000	31.250	0.175	0.103	2.295	0.090	2.292
14	1.750	4.536	2.829	32.500	33.750	0.068	0.038	1.047	0.033	1.045
15	98.250	45.799	13.543	35.000	36.250	0.030	0.017	0.570	0.015	0.569
16	3.500	7.657	3.769	37.500	38.750	0.008	4.001E-4	0.017	3.504E-4	0.017
17	7.000	10.657	4.741	40.000	41.250	0.015	0.008	0.381	0.007	0.381
18	48.250	27.556	9.824	42.500	43.750	0.008	0.005	0.274	0.004	0.273
19	8.500	11.950	5.103	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	2.500	5.536	3.199	0.000	0.000	0.000	0.000	0.000	0.000	0.000
21	2.500	5.243	3.140	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22	8.250	10.657	4.906	0.000	0.000	0.000	0.000	0.000	0.000	0.000
23	2.750	5.536	3.248	0.000	0.000	0.000	0.000	0.000	0.000	0.000
24	14.000	14.243	5.997	0.000	0.000	0.000	0.000	0.000	0.000	0.000
25	52.750	32.385	10.411	0.000	0.000	0.000	0.000	0.000	0.000	0.000
26	162.500	62.113	16.911	0.000	0.000	0.000	0.000	0.000	0.000	0.000
27	1.250	3.828	2.543	0.000	0.000	0.000	0.000	0.000	0.000	0.000
28	33.500	23.142	8.492	0.000	0.000	0.000	0.000	0.000	0.000	0.000
29	135.000	61.234	15.807	0.000	0.000	0.000	0.000	0.000	0.000	0.000
30	29.000	22.435	8.093	0.000	0.000	0.000	0.000	0.000	0.000	0.000

binning



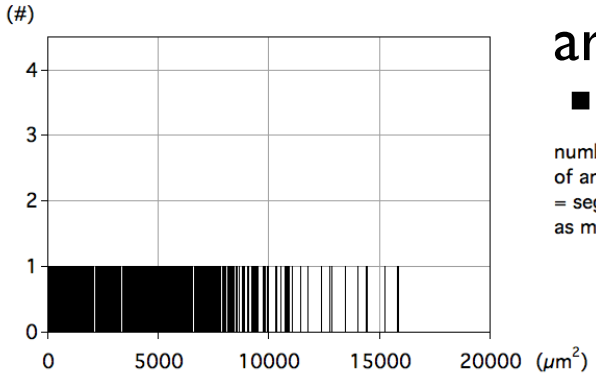




area of circle =
area of segment
(incl. boundary)

diameter of
area equivalent
circle

as is - not binned

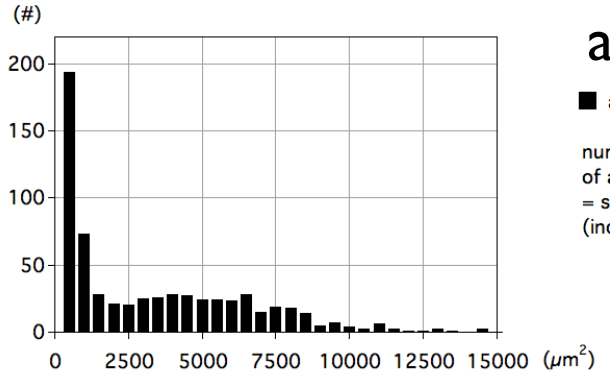


areas

■ #

number (count)
of area
= segment size
as measured

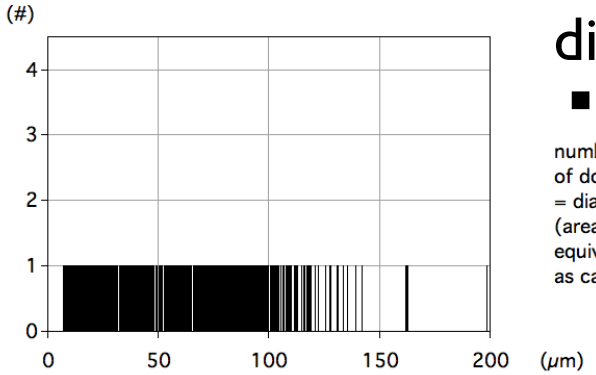
binned - histogram



areas

■ area corr (µm2)

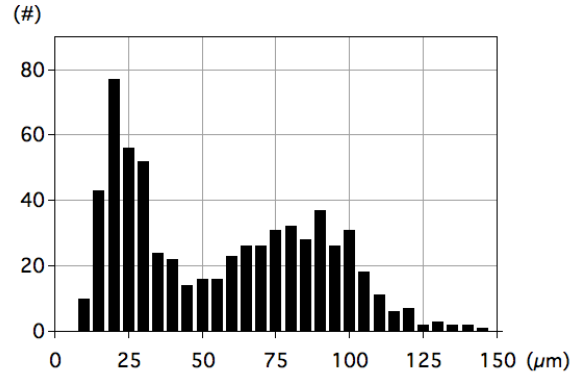
number frequency
of area
= segment size
(incl. boundary)



diameters

■ #

number (count)
of dcorr
= diameter of
(area+boundary)
equivalent circle
as calculated



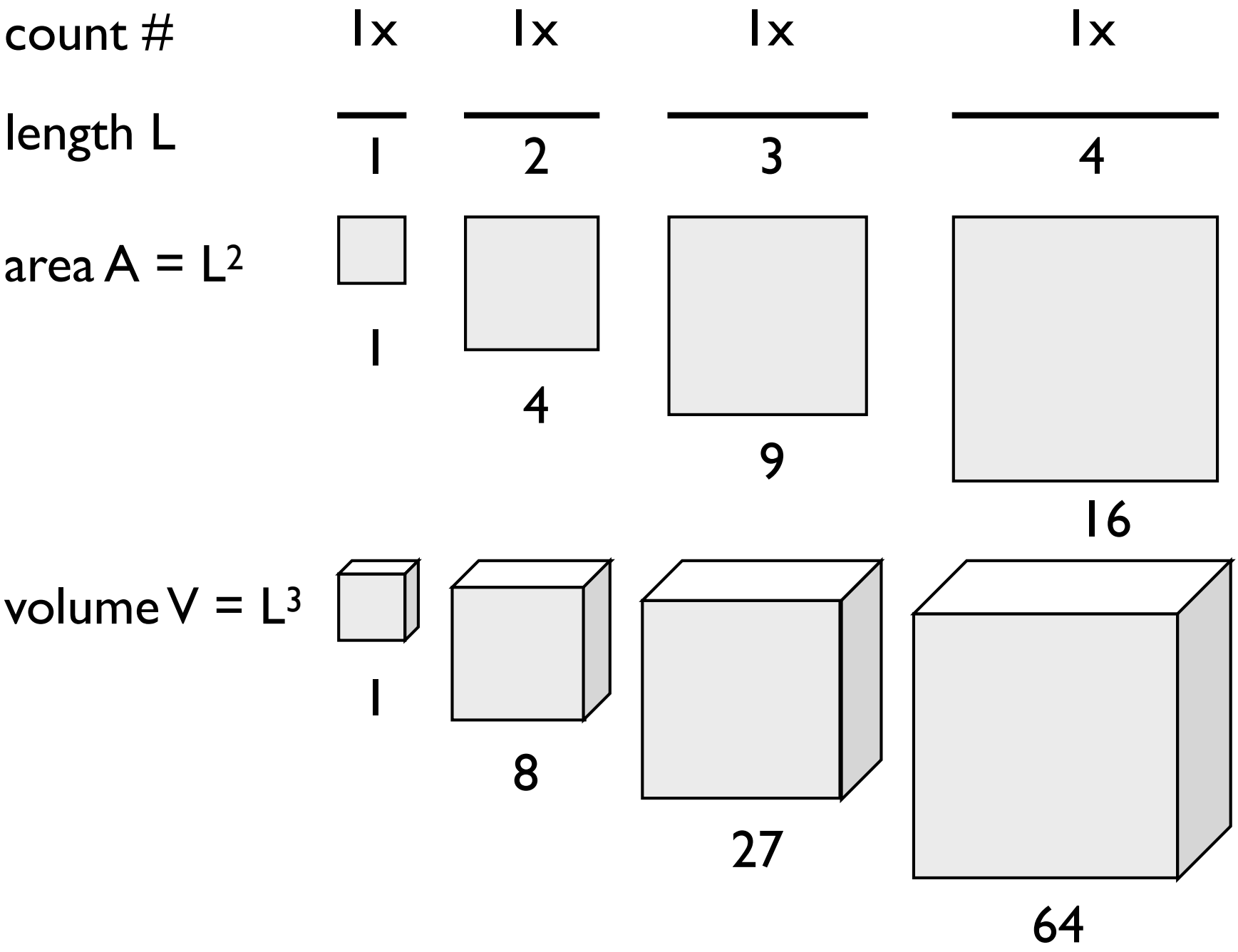
diameters

■ h(dcorr)

number frequency
of dcorr

(= diameter of area
equivalent cirle)

weighting


count #
1x
1x
1x
1x
length L
1
2
3
4
area A = L²
1
4
9
16
volume V = L³
1
8
27
64

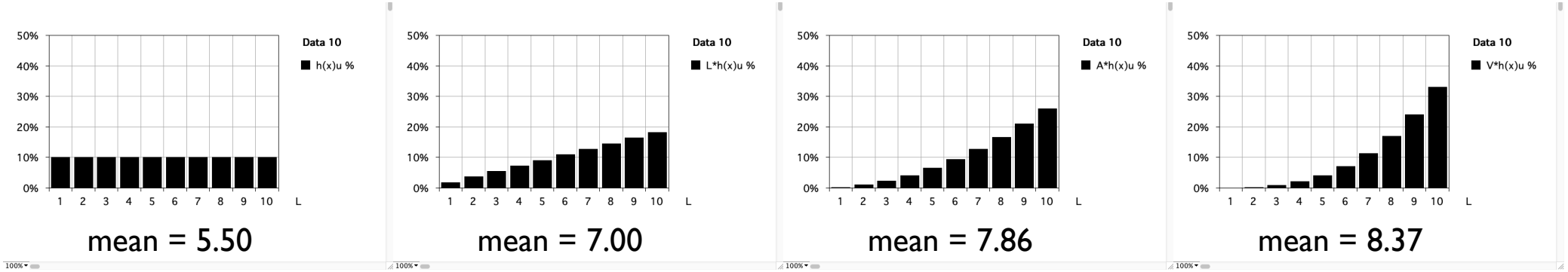
from histogram to log-log functions

number weighted

length weighted

area weighted

volume weighted



mean = 5.50

mean = 7.00

mean = 7.86

mean = 8.37

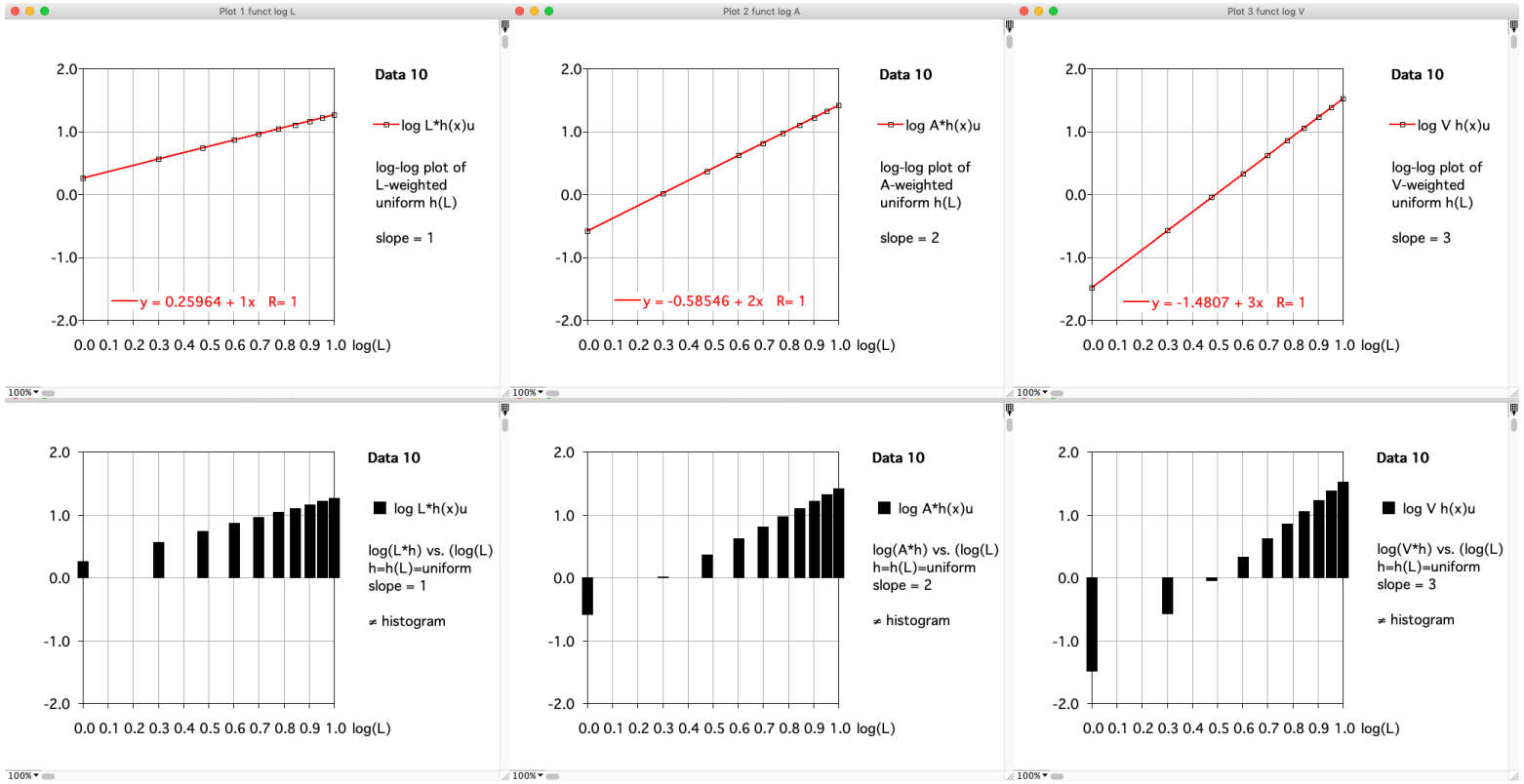
lin-lin

not a histogram !!

log-log

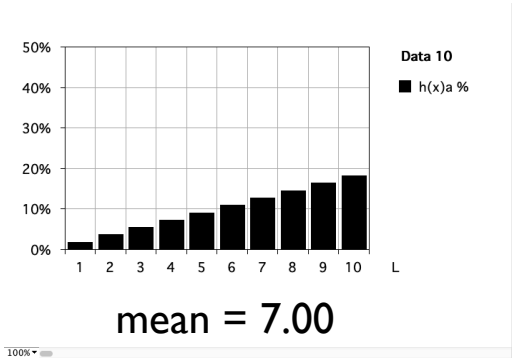
not a histogram !!

log-log



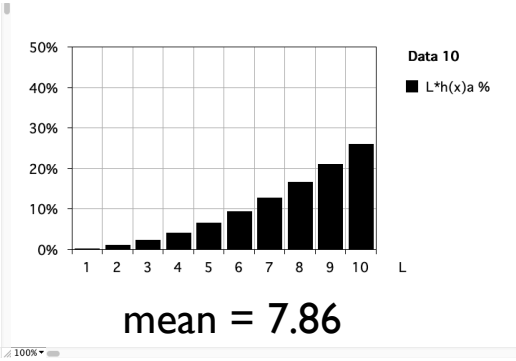
from histogram to log-log functions

number weighted



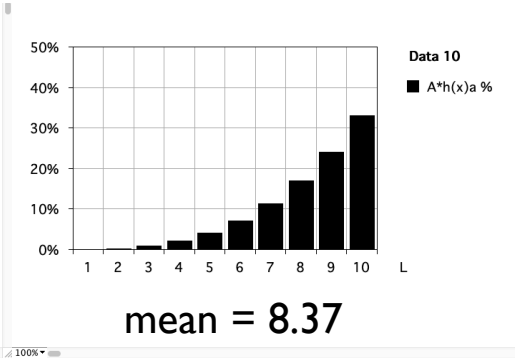
mean = 7.00

length weighted



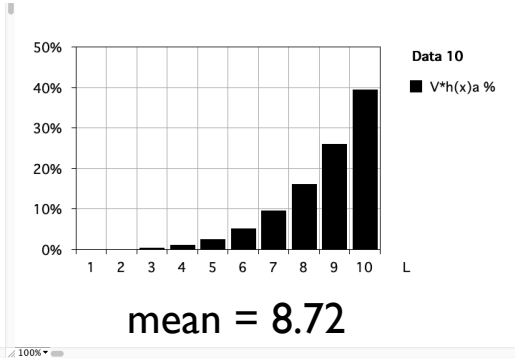
mean = 7.86

area weighted



mean = 8.37

volume weighted

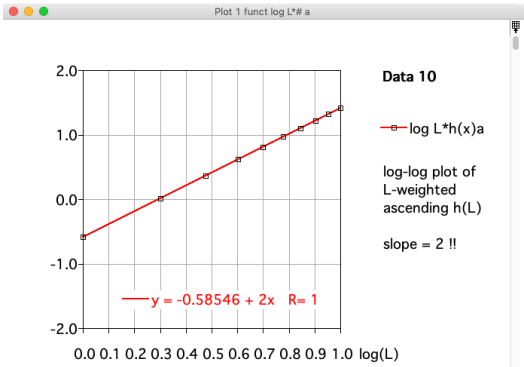


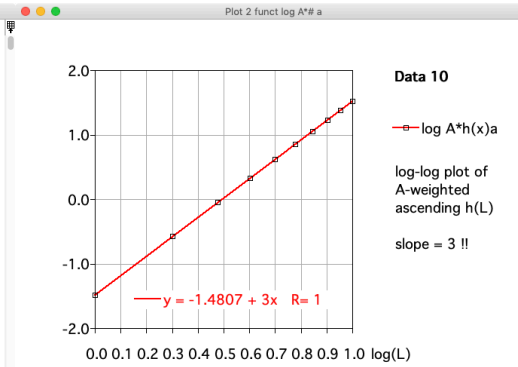
mean = 8.72

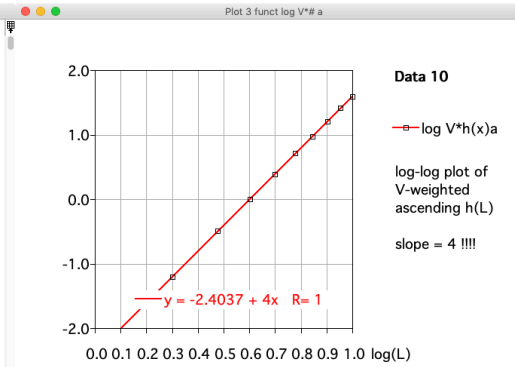
lin-lin

not a histogram !!

log-log

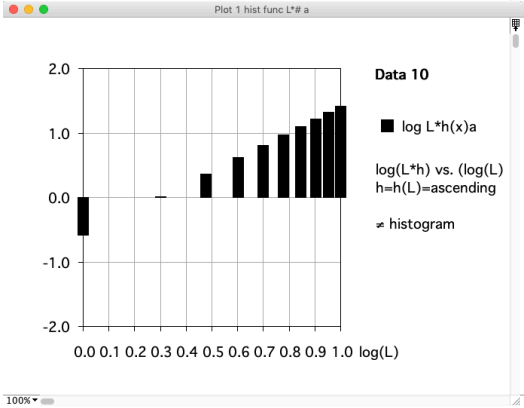


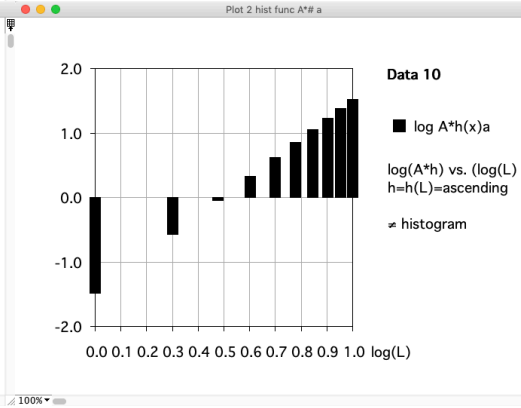


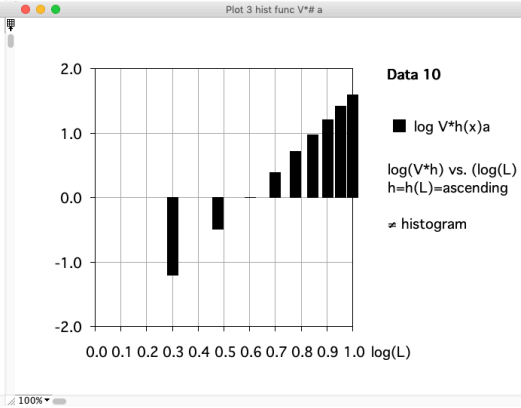


not a histogram !!

log-log

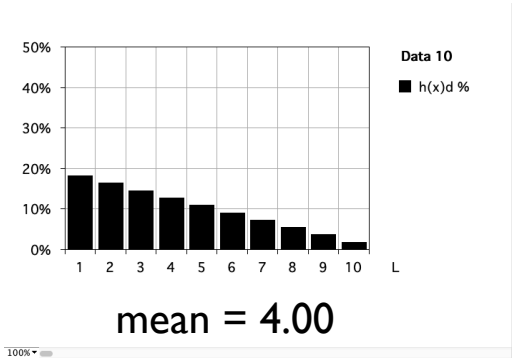






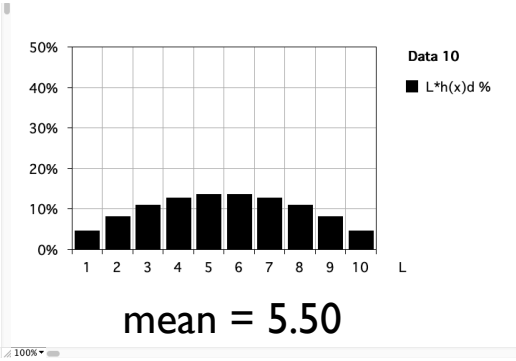
from histogram to log-log functions

number weighted



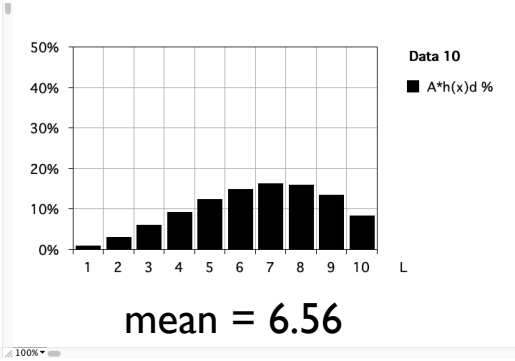
mean = 4.00

length weighted



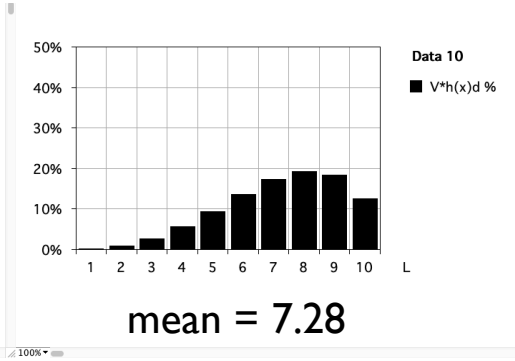
mean = 5.50

area weighted



mean = 6.56

volume weighted

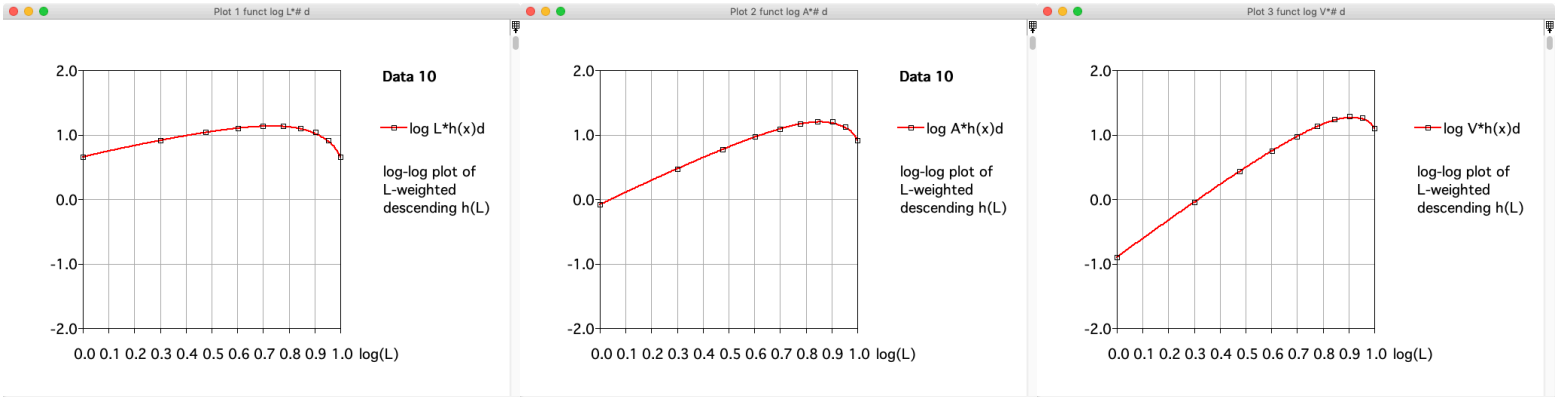


mean = 7.28

lin-lin

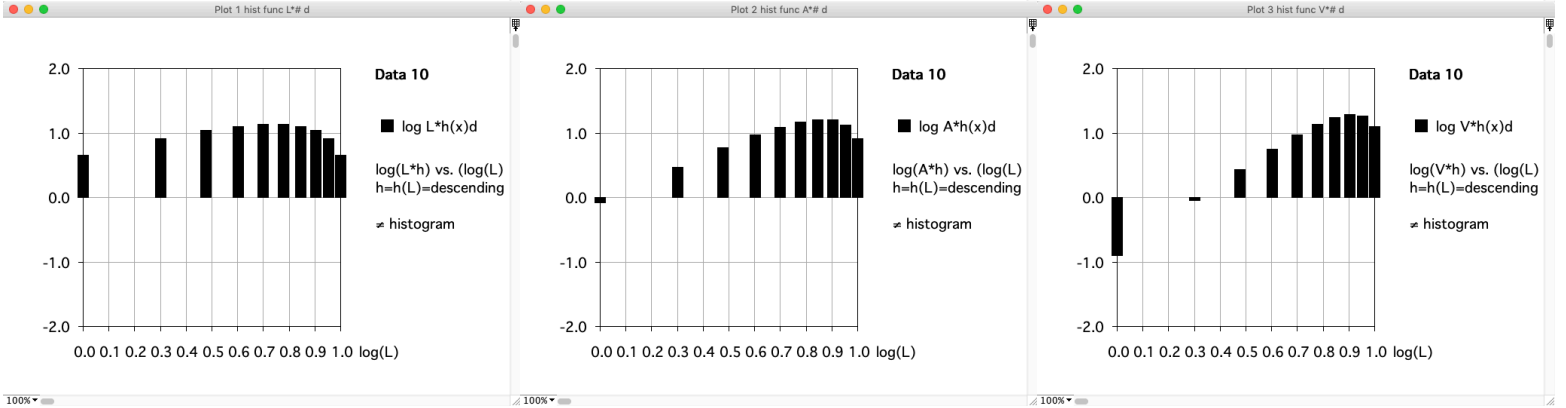
not a histogram !!

log-log



not a histogram !!

log-log



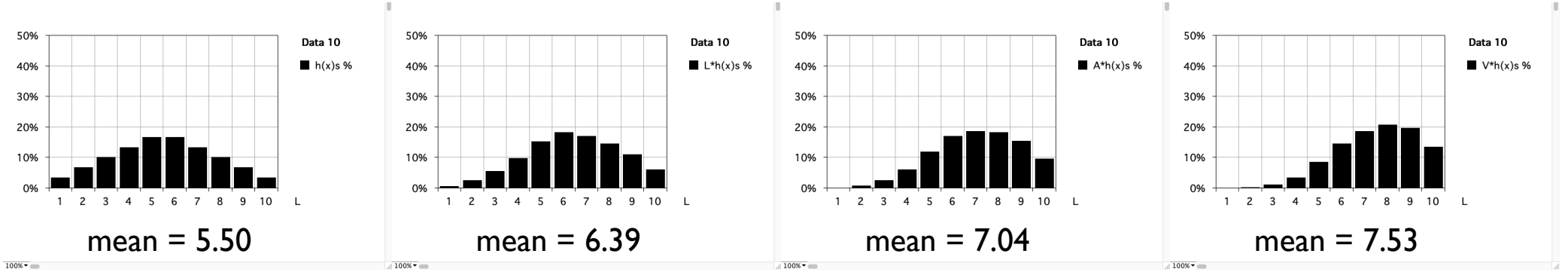
from histogram to log-log functions

number weighted

length weighted

area weighted

volume weighted



mean = 5.50

mean = 6.39

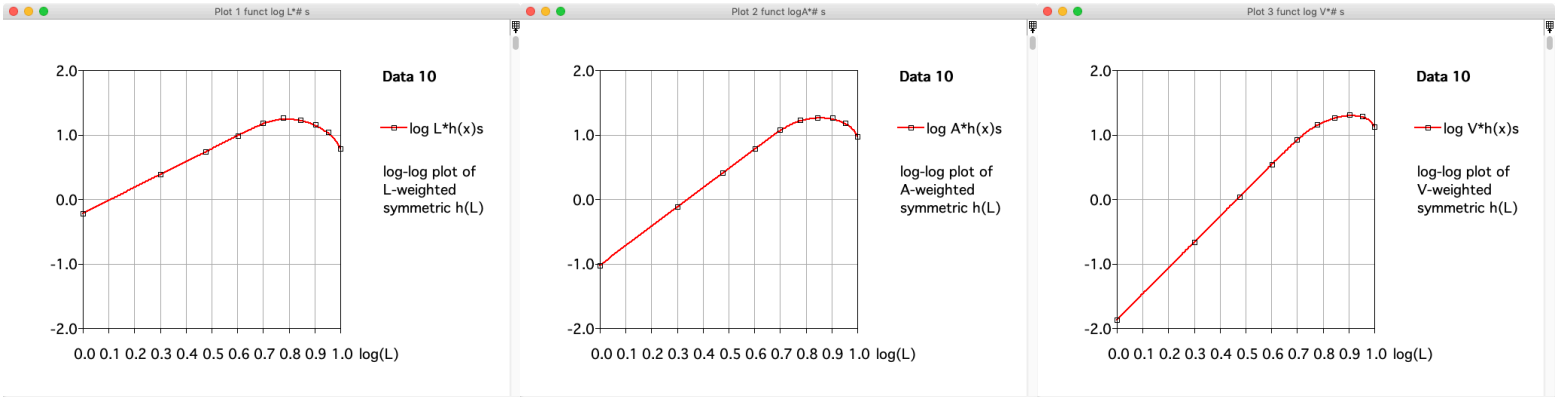
mean = 7.04

mean = 7.53

lin-lin

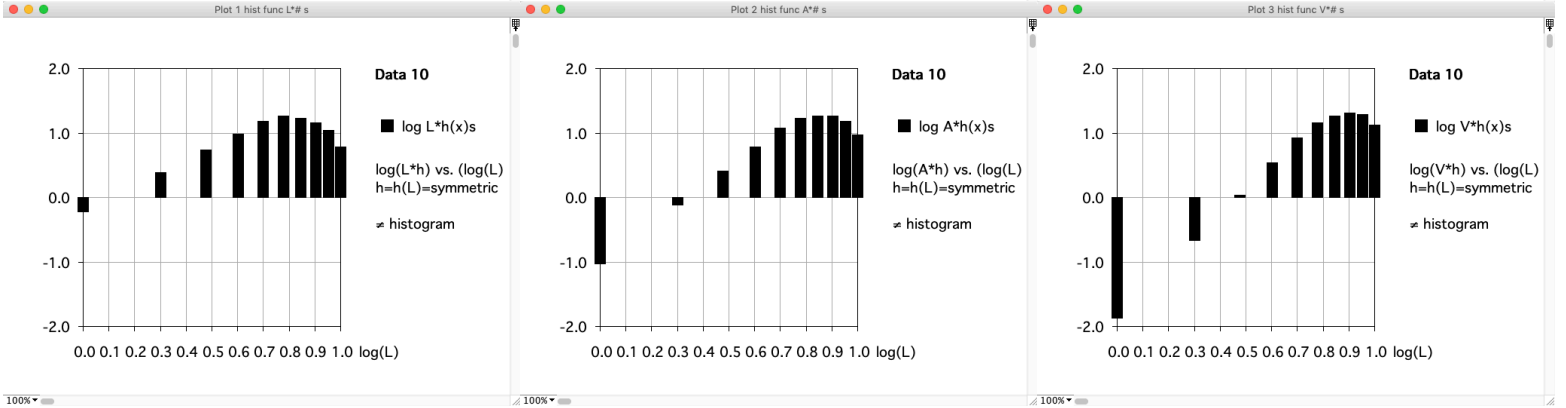
not a histogram !!

log-log



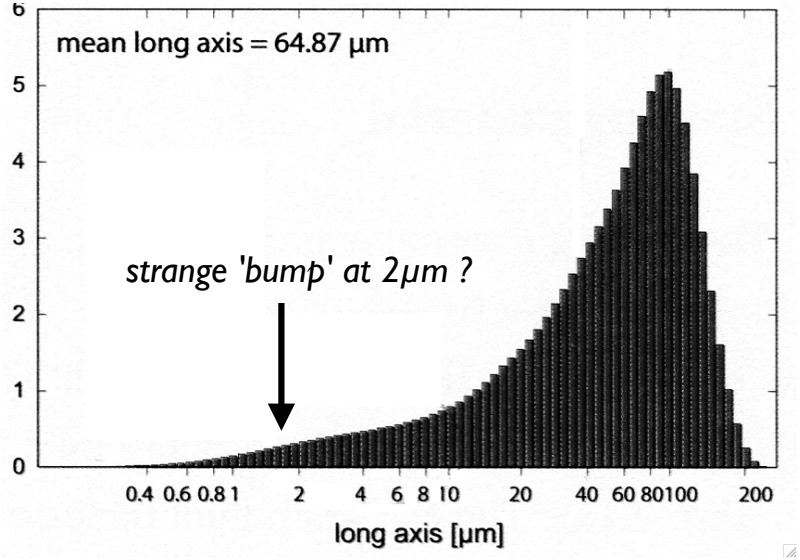
not a histogram !!

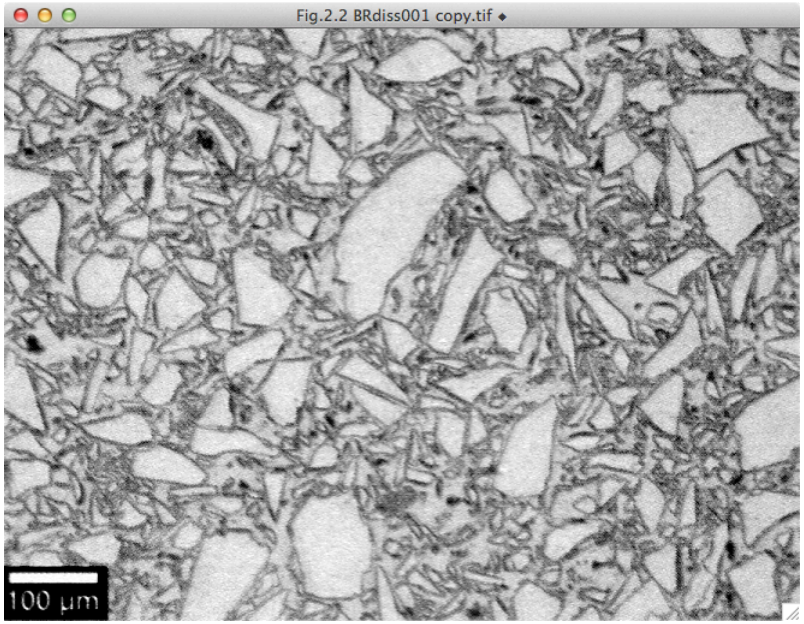
log-log



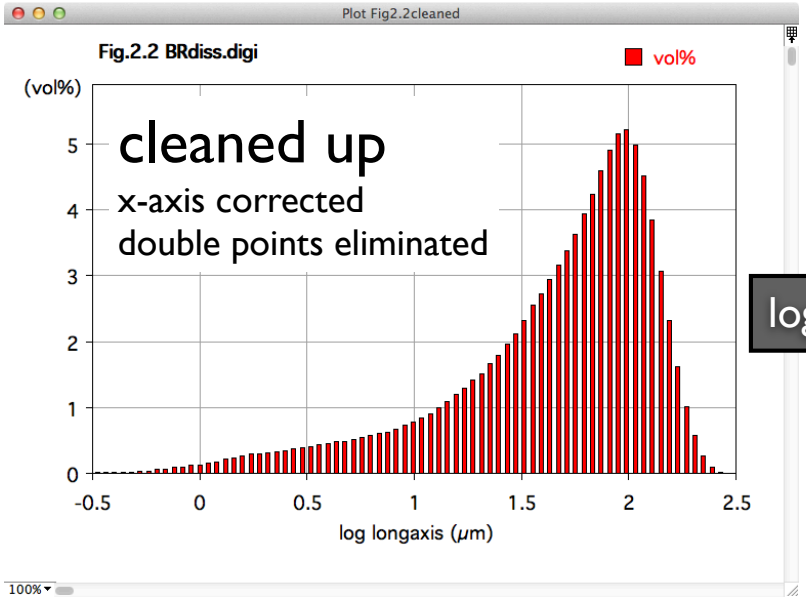
convert log histograms to linear

mean = 65µm mode = 100µm ??

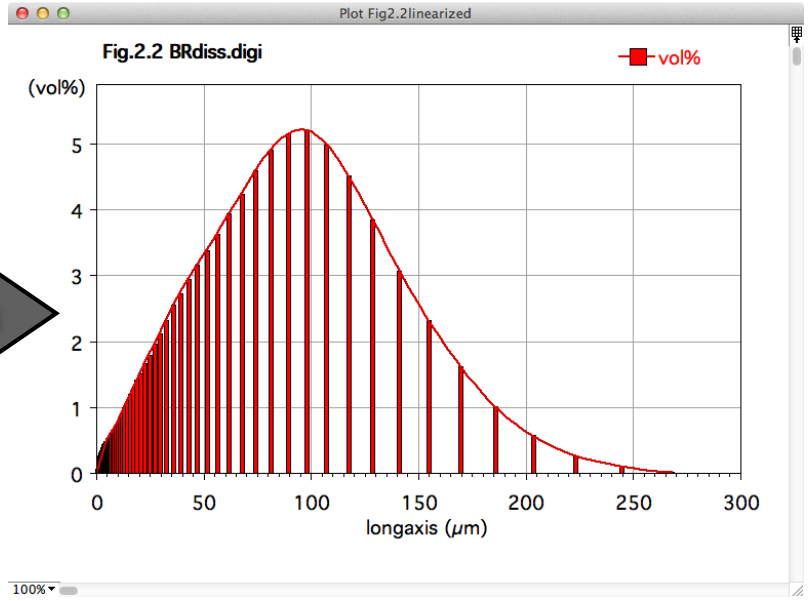




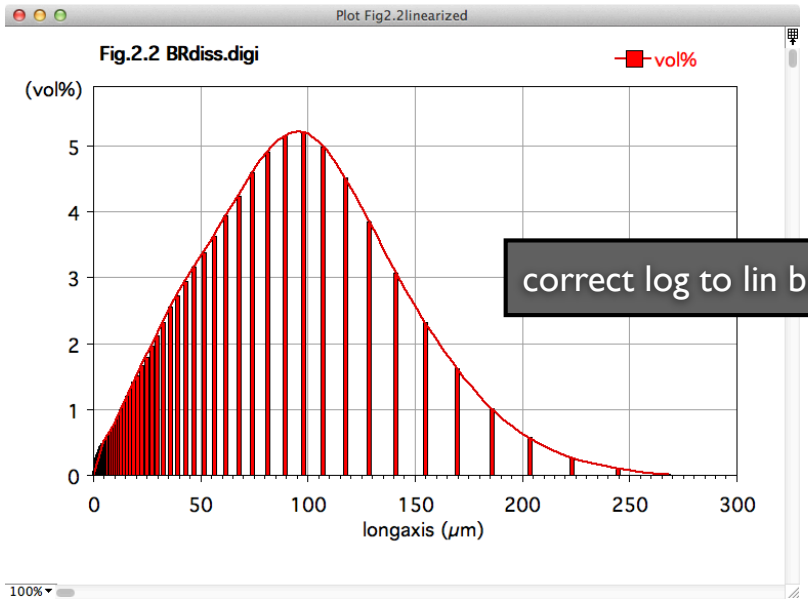
Betti's qtz gouge



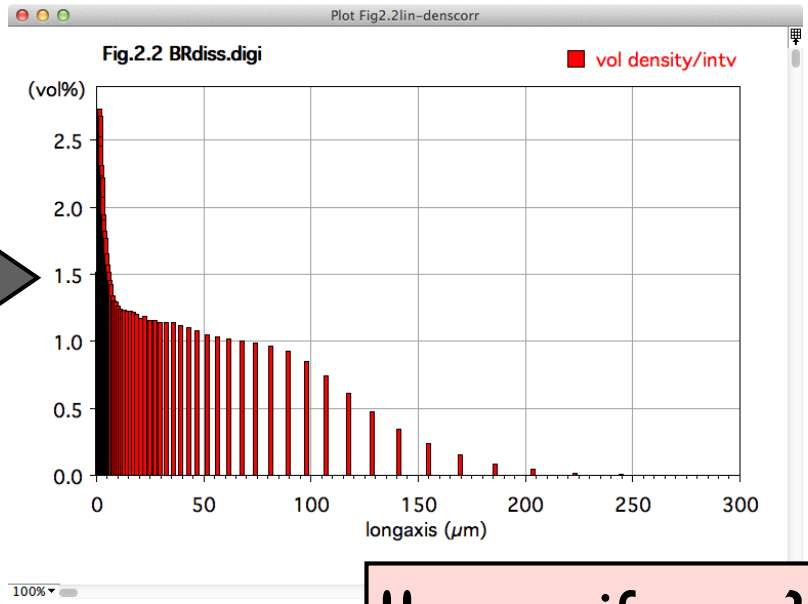
log to lin x-axis



vol% (log(size)) to vol% (size)

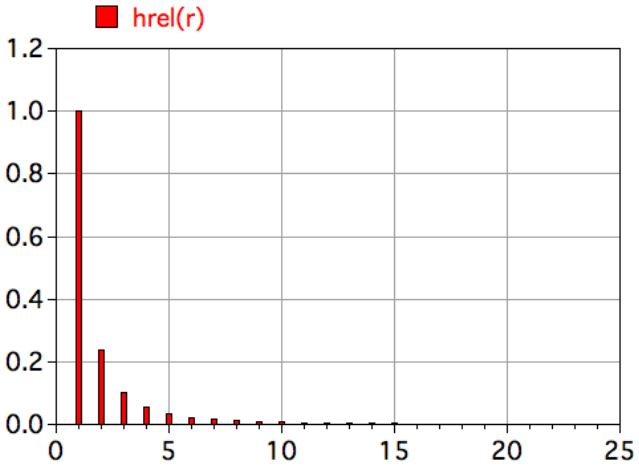


correct log to lin bin width



!! = uniform ? fractal ??

h(a) recalculated from w%(log(a))

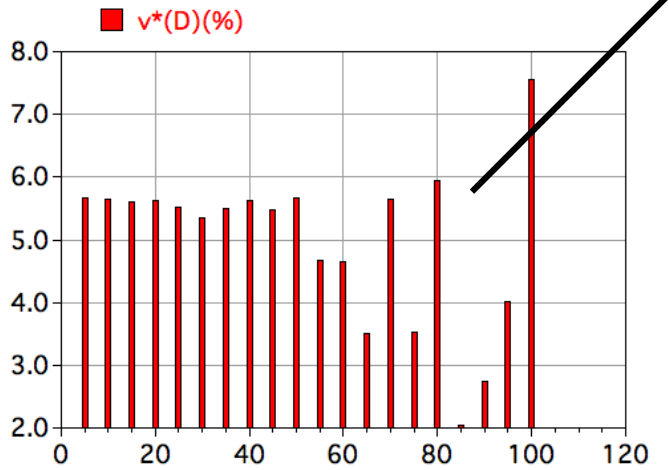


a = 2D long axis

r(=a/2)

(CalSecDis)

h(A) recalculated from h(a)



A = 3D long axis

R(=A/2)

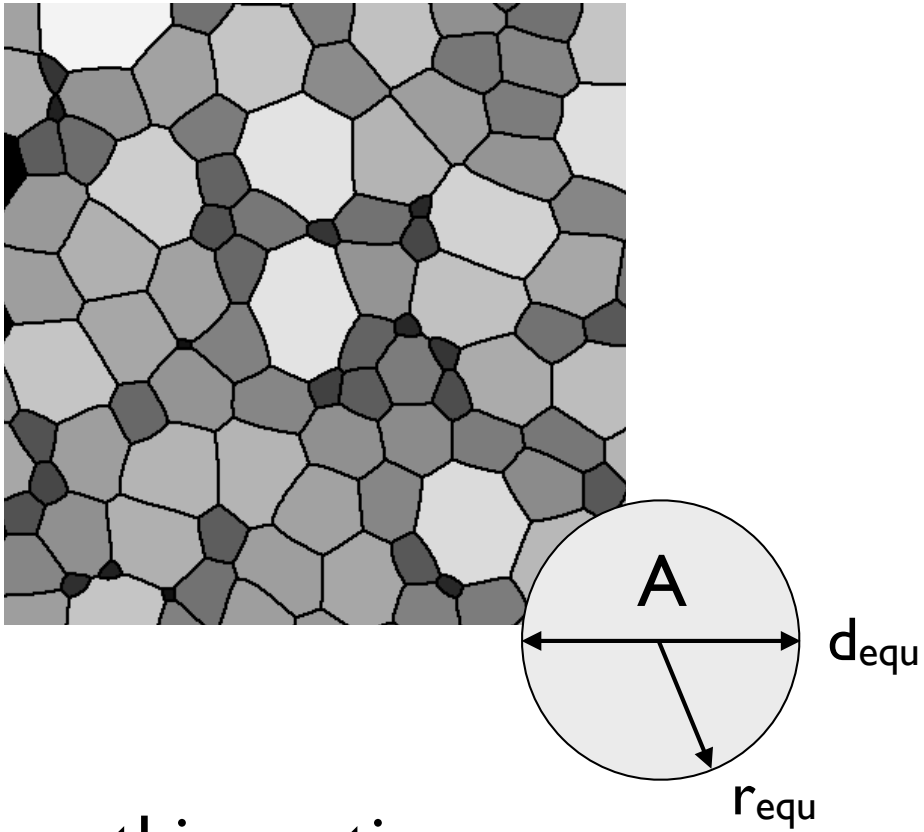
(stripD)

3
2D grain size

defining grain size

- measures for grain size
- finding the correct diameter

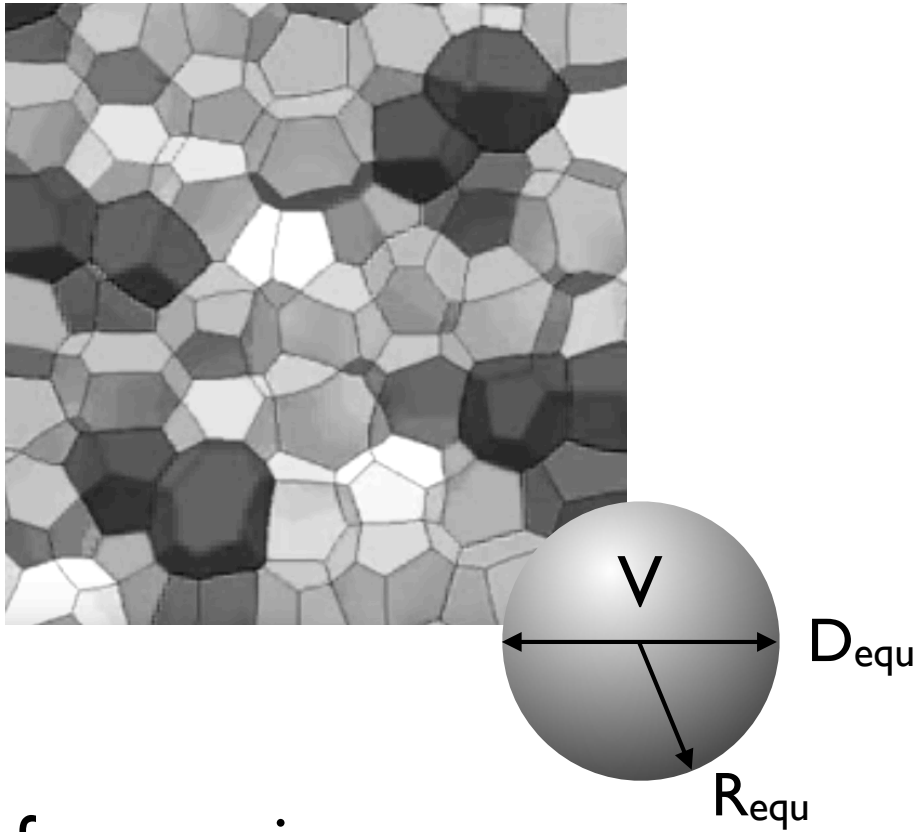
diameter - a measure for grain size



on thin section
diameter of area

area of circle
$A = \pi \cdot r^2$

$\Rightarrow\ d = 2 \cdot \sqrt[2]{(A/\pi)}$

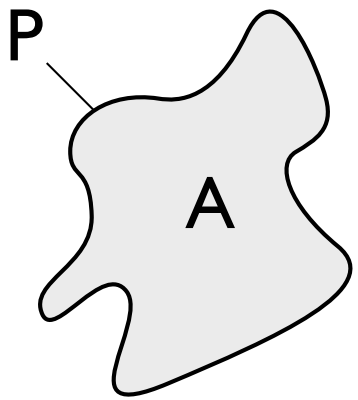


from grains
diameter of volume

volume of sphere
$V = 4\pi/3 \cdot r^3$

$\Rightarrow\ D = 2 \cdot \sqrt[3]{(3V/(4\pi))}$

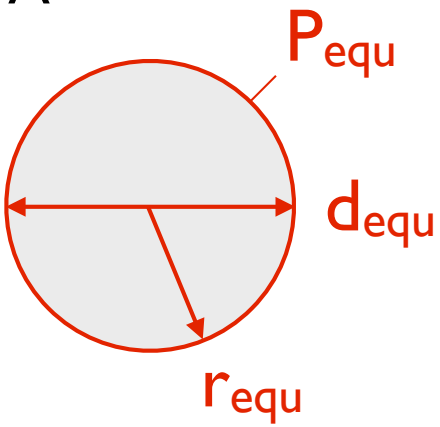
alternative measures



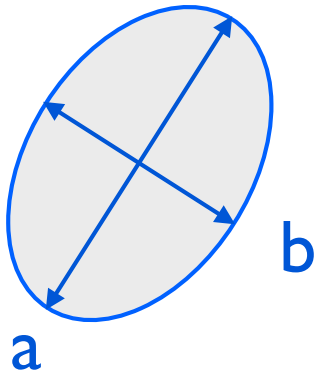
P perimeter,
A area of shape

use one of these

same A



same A

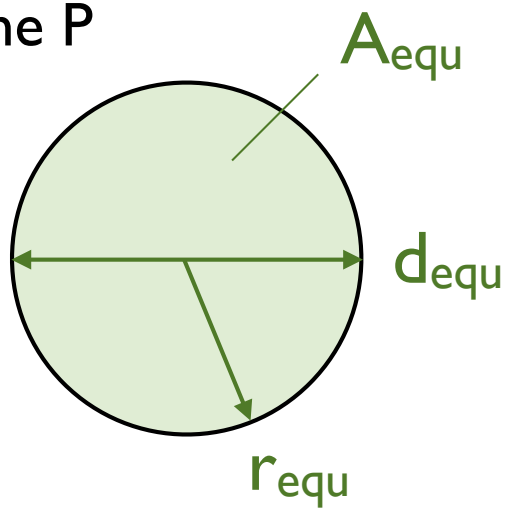


r_{equ} radius,
d_{equ} diameter
of area equivalent circle

a longest diameter
b shortest diameter
of best fit ellipse

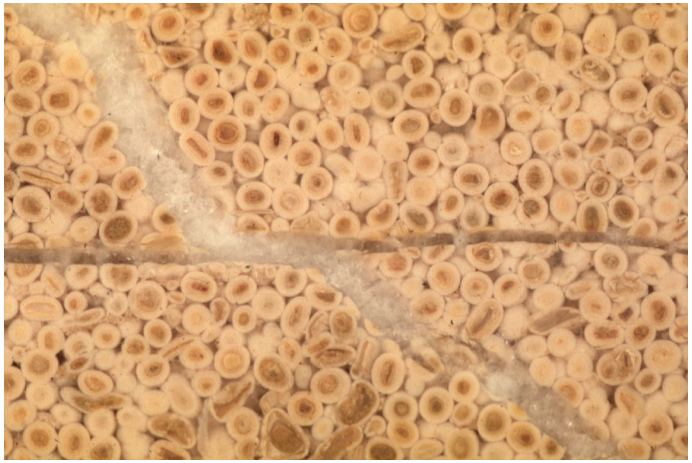
don't use

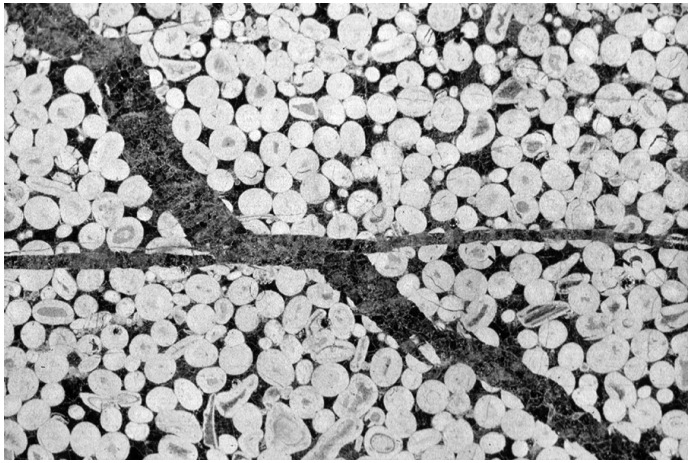
same P



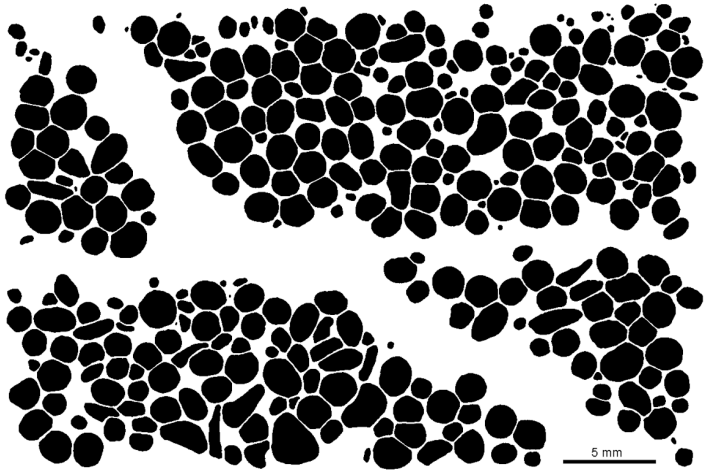
r_{equ} = P / 2π = radius,
d_{equ} = P / π = diameter
of perimeter equivalent circle

different diameters - isometric case

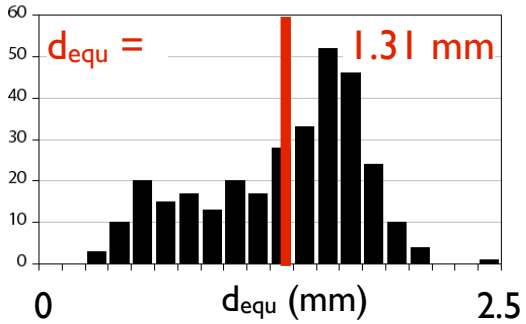


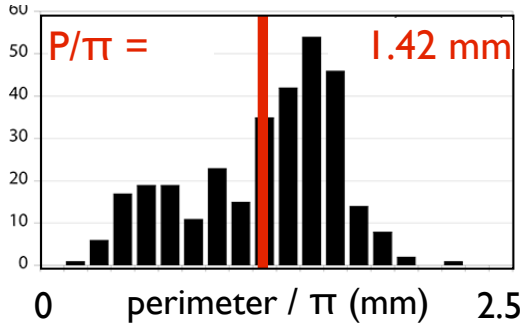


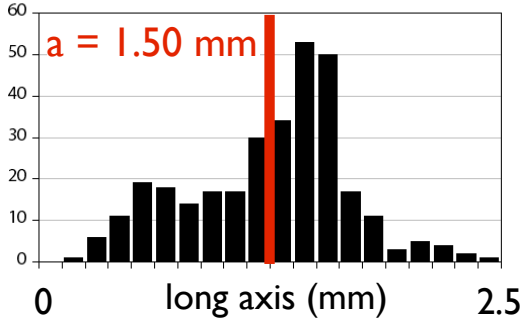
Oolithic limestone, polished surface (top)
acetate replica of etched surface (bottom)
Hauptrogenstein, Jura Mts., Switzerland,
Gregor Strub, unfinished PhD thesis, Basel Univ.

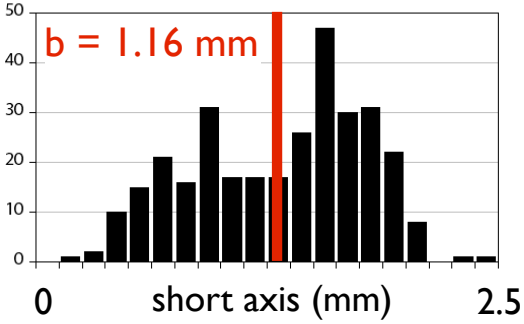


mean grain sizes

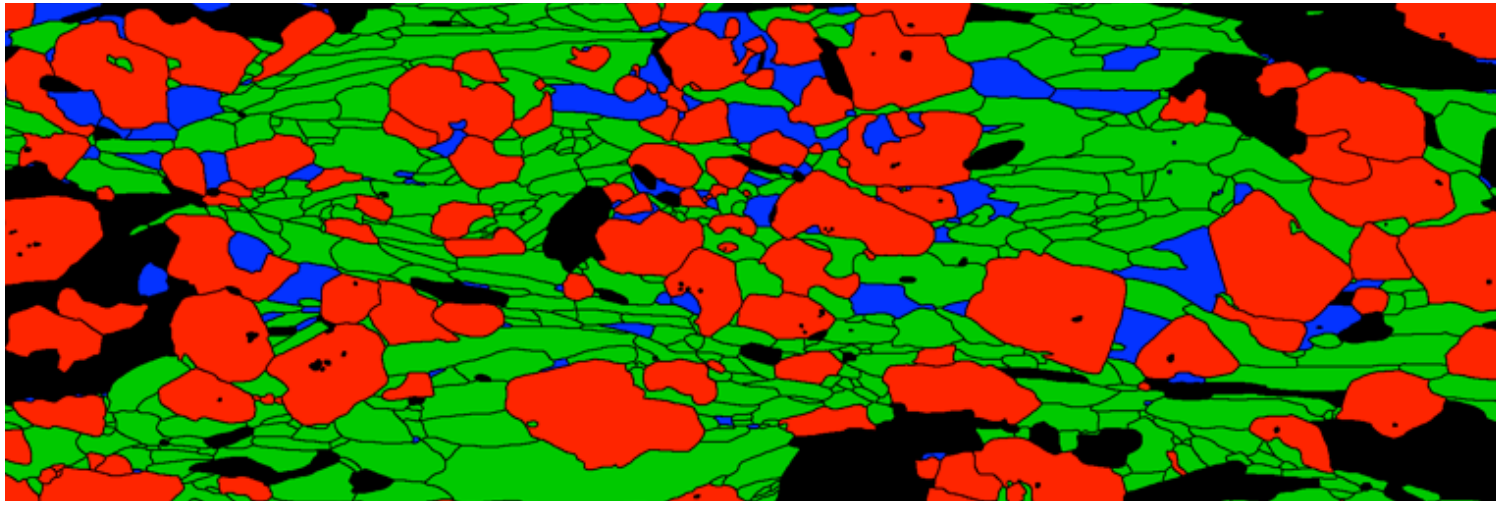






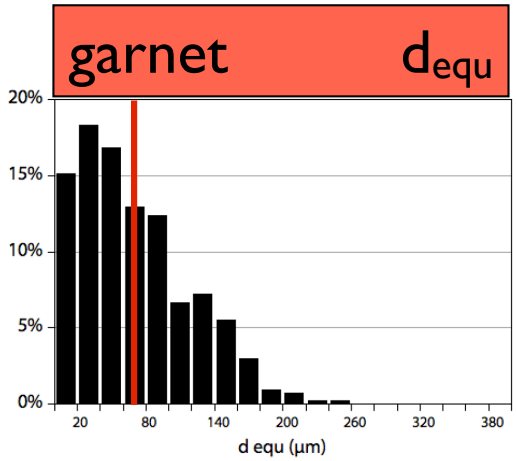


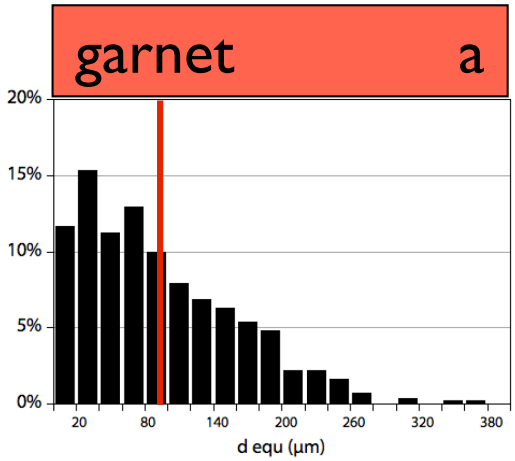
different diameters - anisometric case

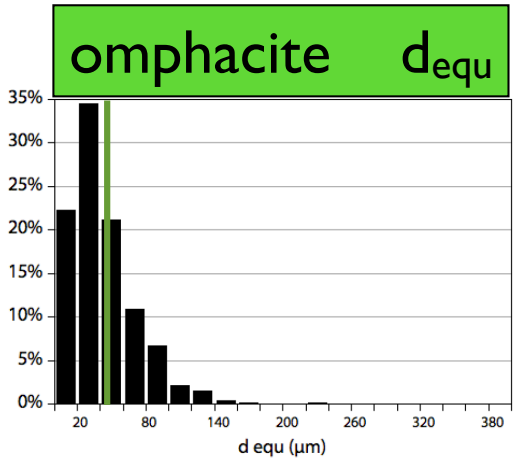


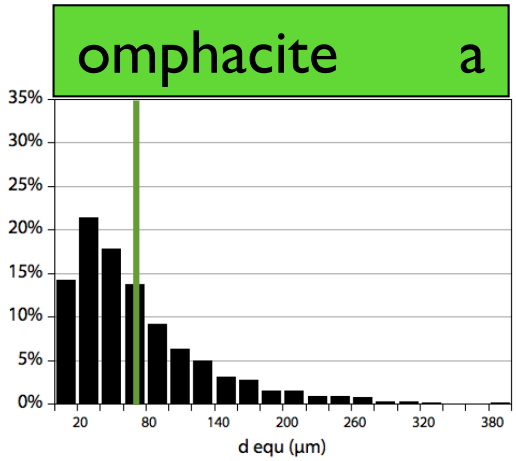
500 µm

Data from Ane Kongsro Finstad (2017) Master thesis in Geology, Tromsø





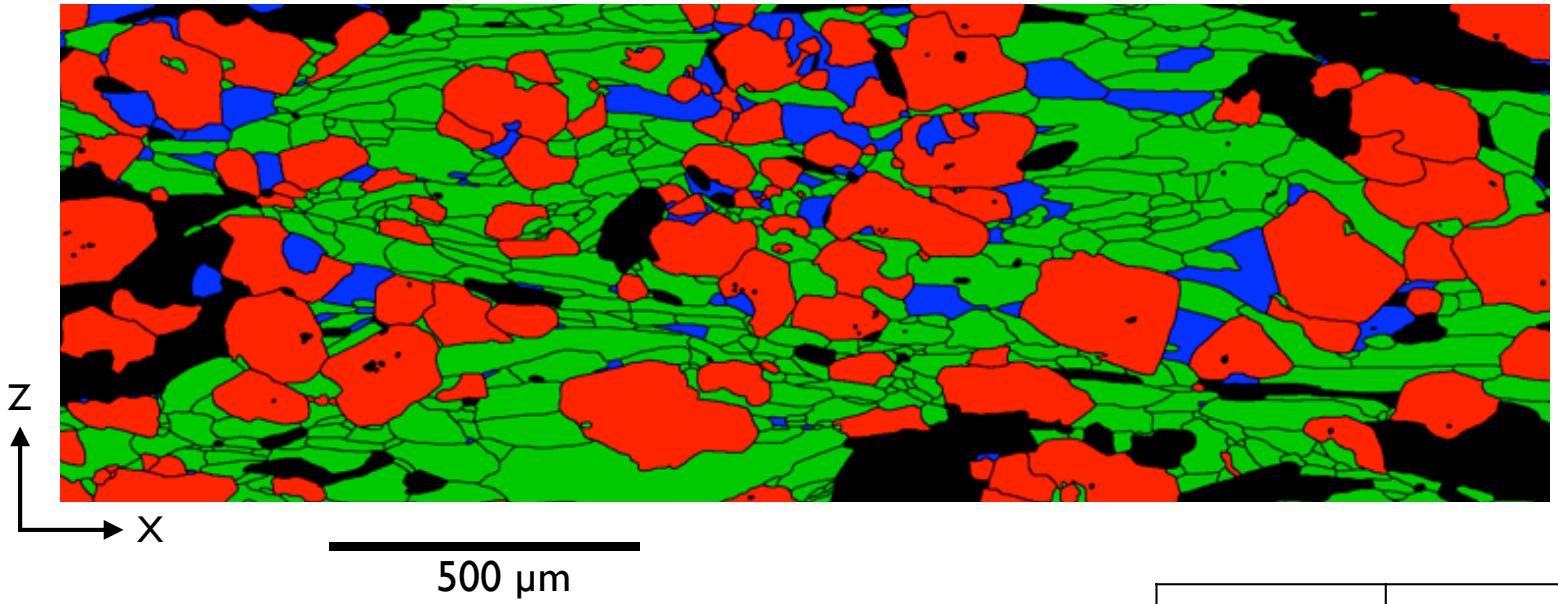


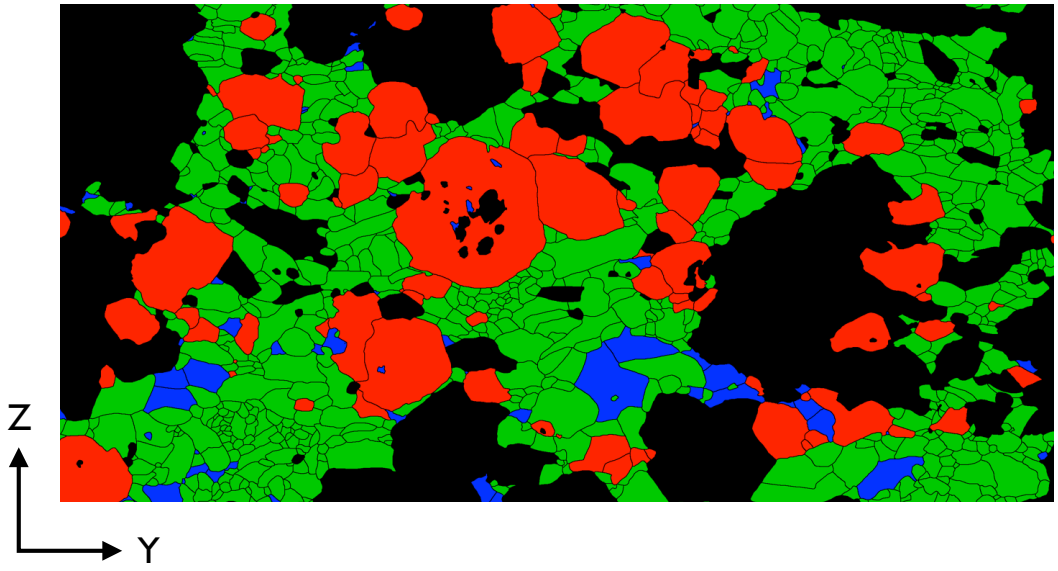


Mean	**69.8**	**93.3**
Median	59.6	78.7
RMS	84.7	115.3
Std Deviation	48.2	67.7

Mean	**42.7**	**74.2**
Median	35.4	55.3
RMS	51.8	99.9
Std Deviation	29.4	67.0

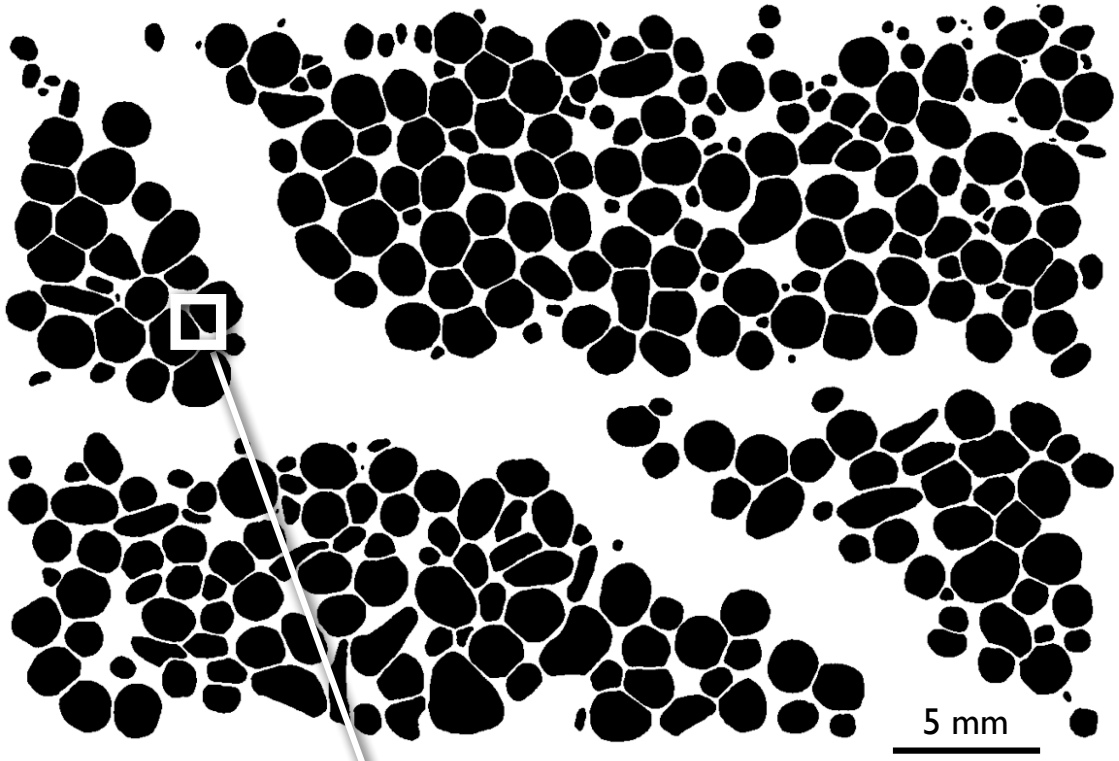
grain size(s) on orthogonal sections

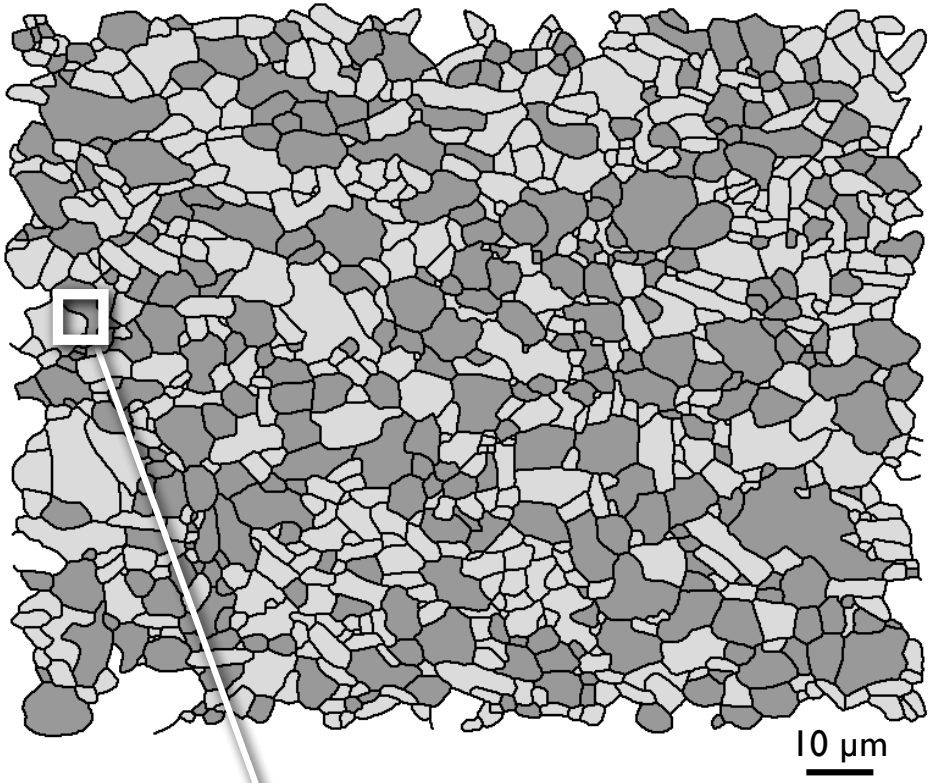


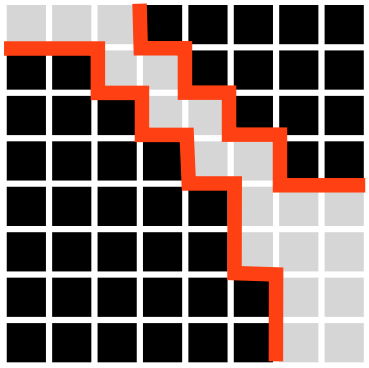


mineral	section	2D mean (µm)	no. of grains
garnet	X-Z	69.8	541
	Y-Z	49.1	94
omphacite	X-Z	42.7	1727
	Y-Z	25.8	759
quartz	X-Z	35.8	586
	Y-Z	21.0	78

true area of segments





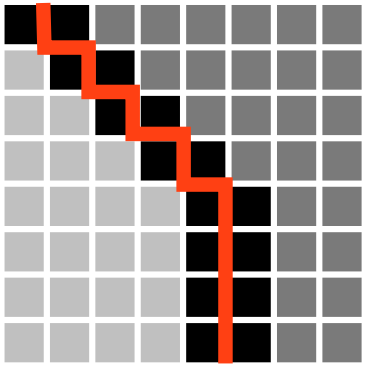


segment boundary

matrix pixels do not
belong to segments

no. of pixels = area of segment

$d_{equ} = \sqrt{(A / \pi)}$



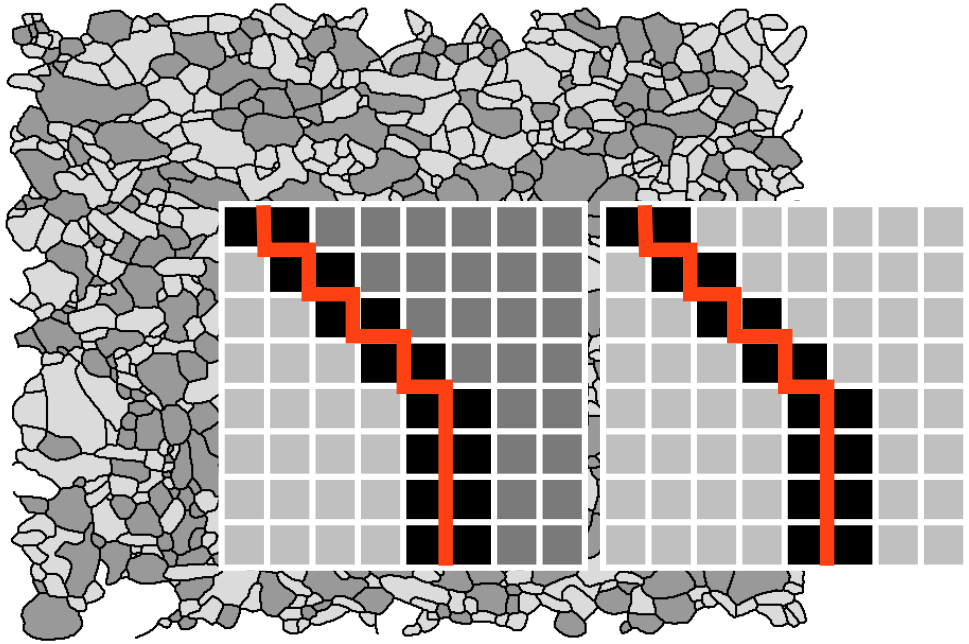
segment boundary

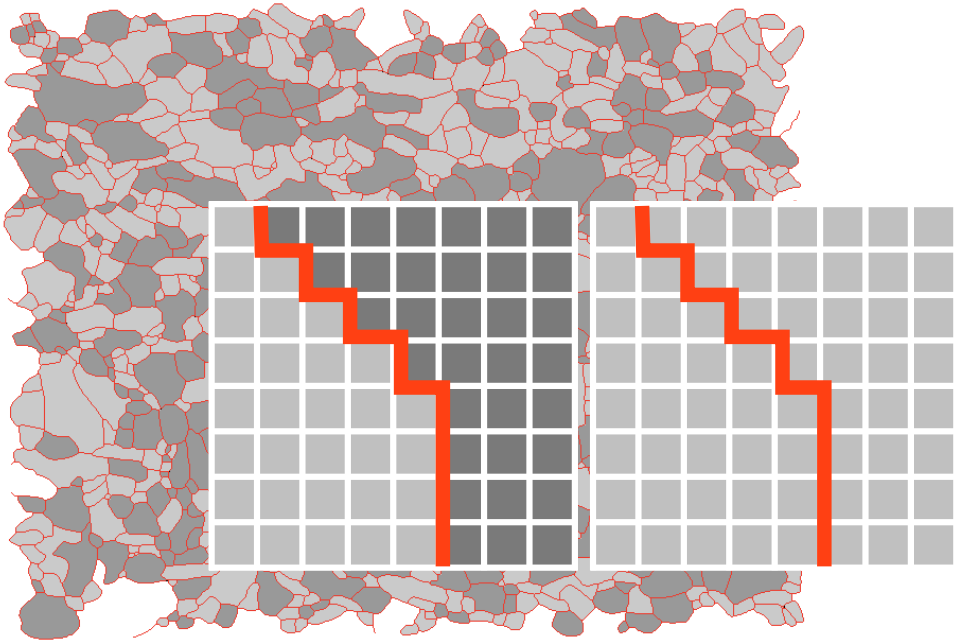
grain boundary pixels
belong to segments

no. of pixels < area of segment

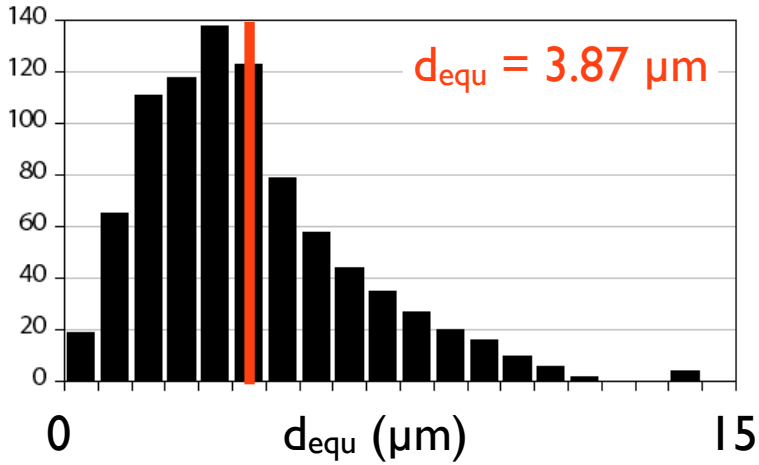
$d_{equ\ corr} = \sqrt{((A+P) / \pi)}$
only if image is scaled to pixel !

reconstructing the segment area



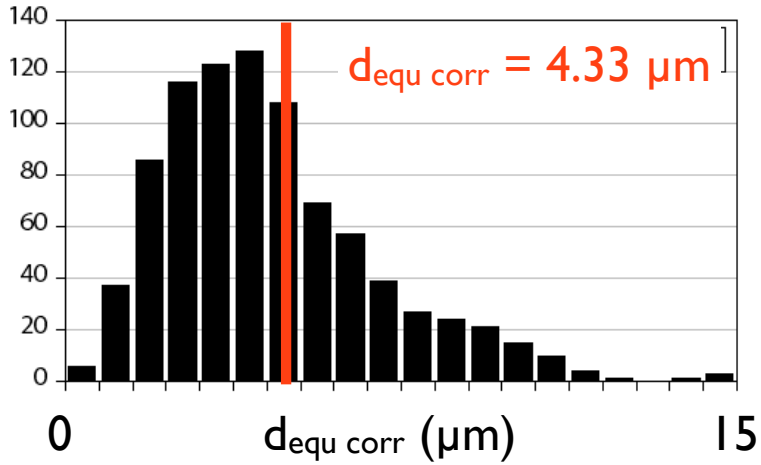


calcite & anhydrite



10 µm

calcite & anhydrite

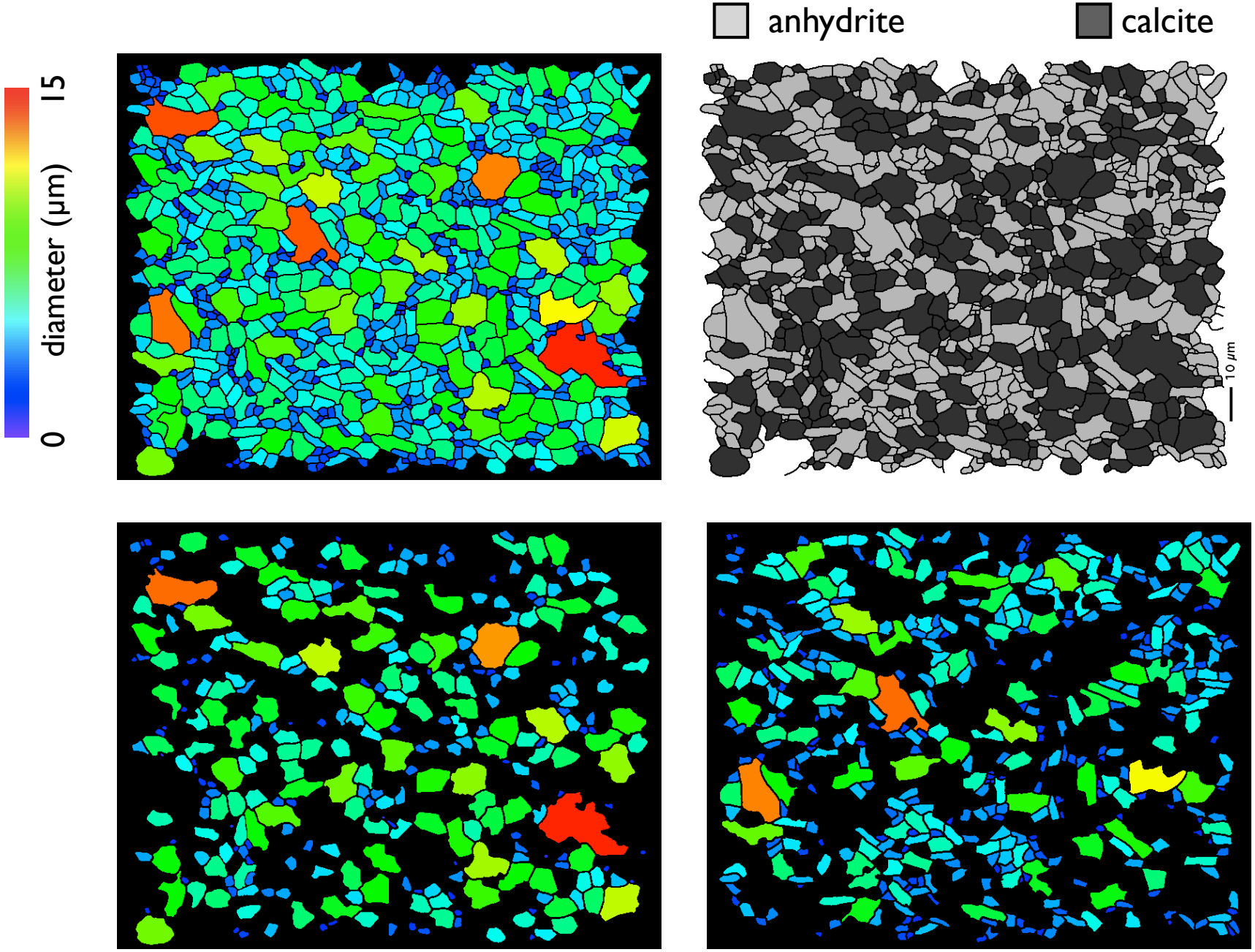


Heilbronner, R. and Bruhn, D (1998).
The influence of three-dimensional grain size distributions on the rheology of polyphase rocks. J. Structural Geology, 20: 695-707.

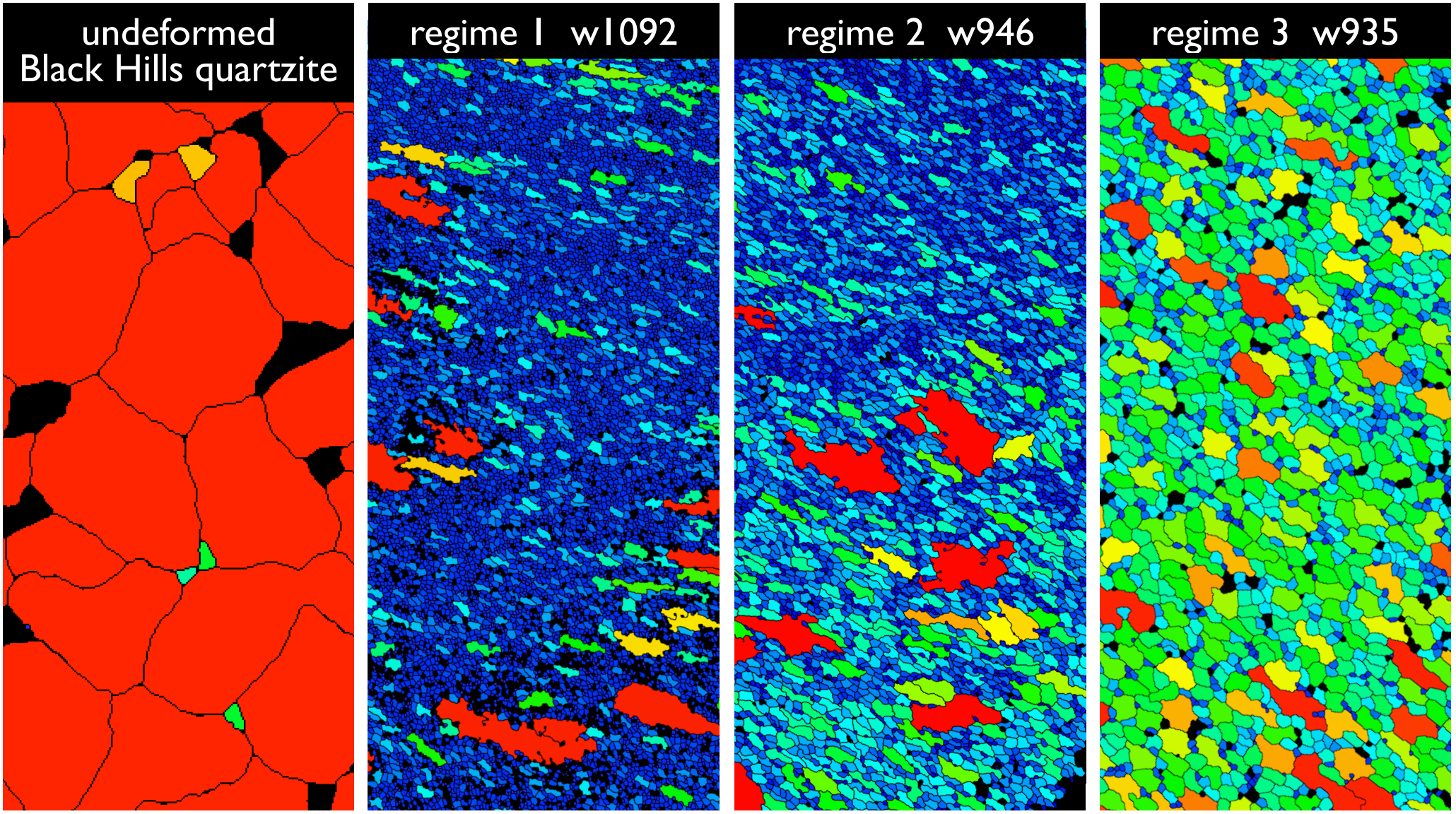
grain size mapping

- visualizing grain size
- area weighted grain size

grain size mapping



recrystallized grain size ... gradients



Heilbronner, R. & Kilian, R. (2017). The grain size(s) of Black Hills Quartzite deformed in the dislocation creep regime. Solid Earth, 8, 1071–1093, 2017, doi.org/10.5194/se-8-1071-2017.

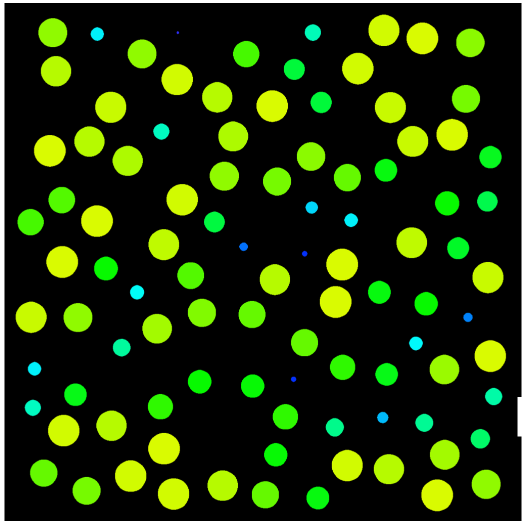
4
3D grain size

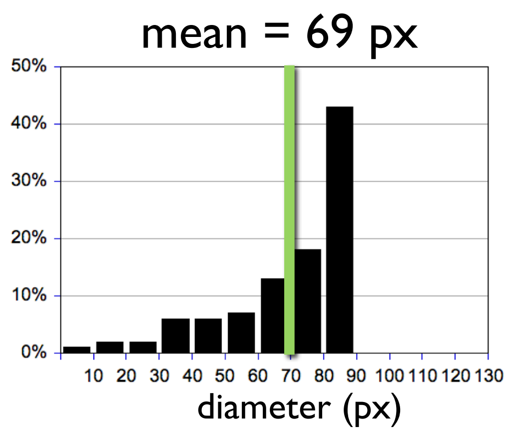
why 3D ?

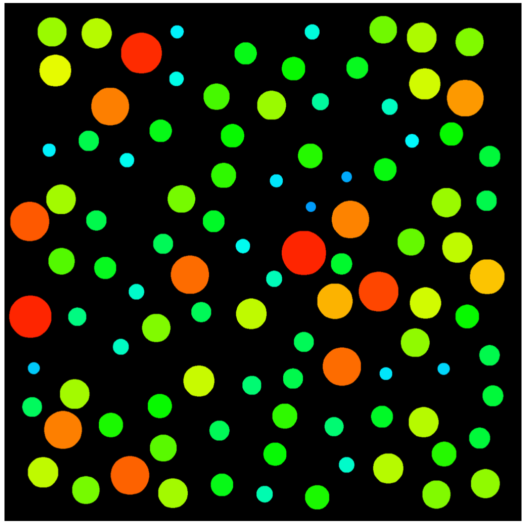
- the famous correction factor

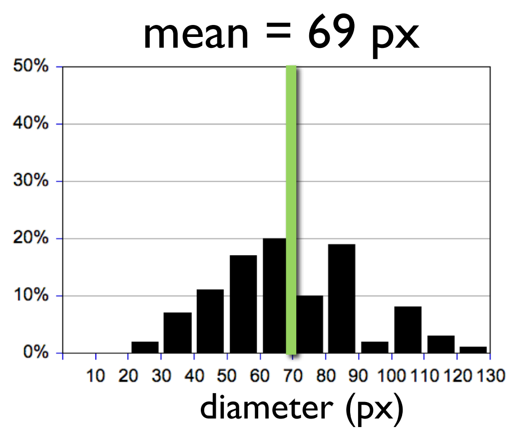
why 3D grain size ?!

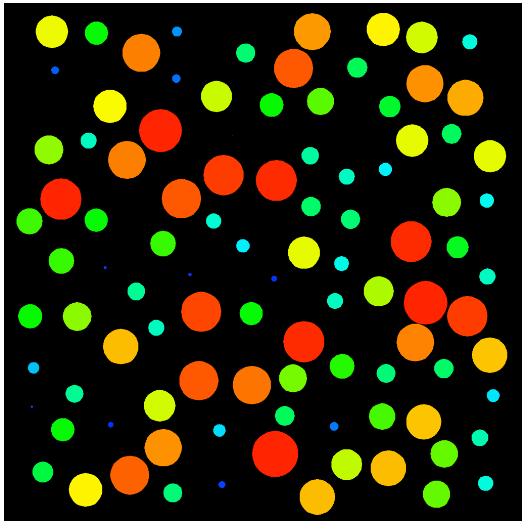
100 random sectional circles calculated for 4 different distributions of spheres

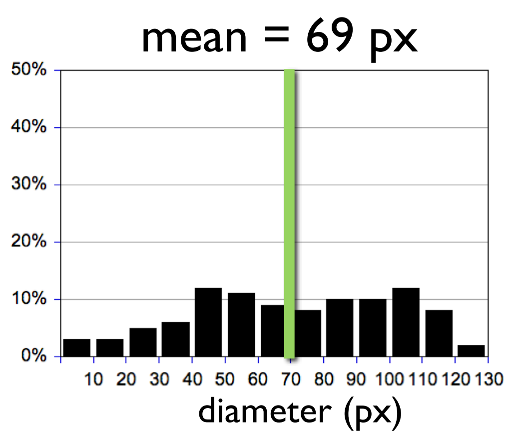


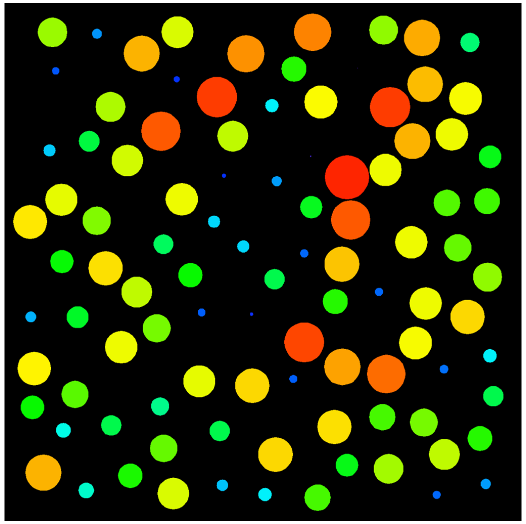


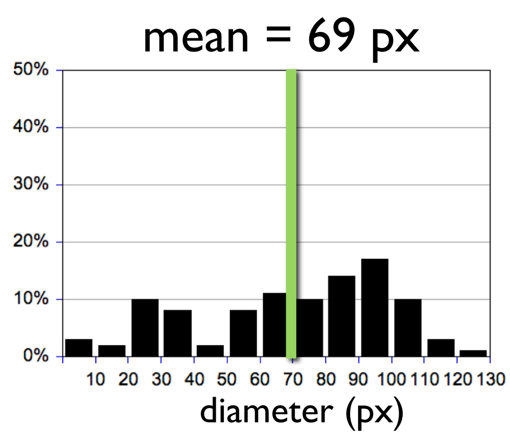




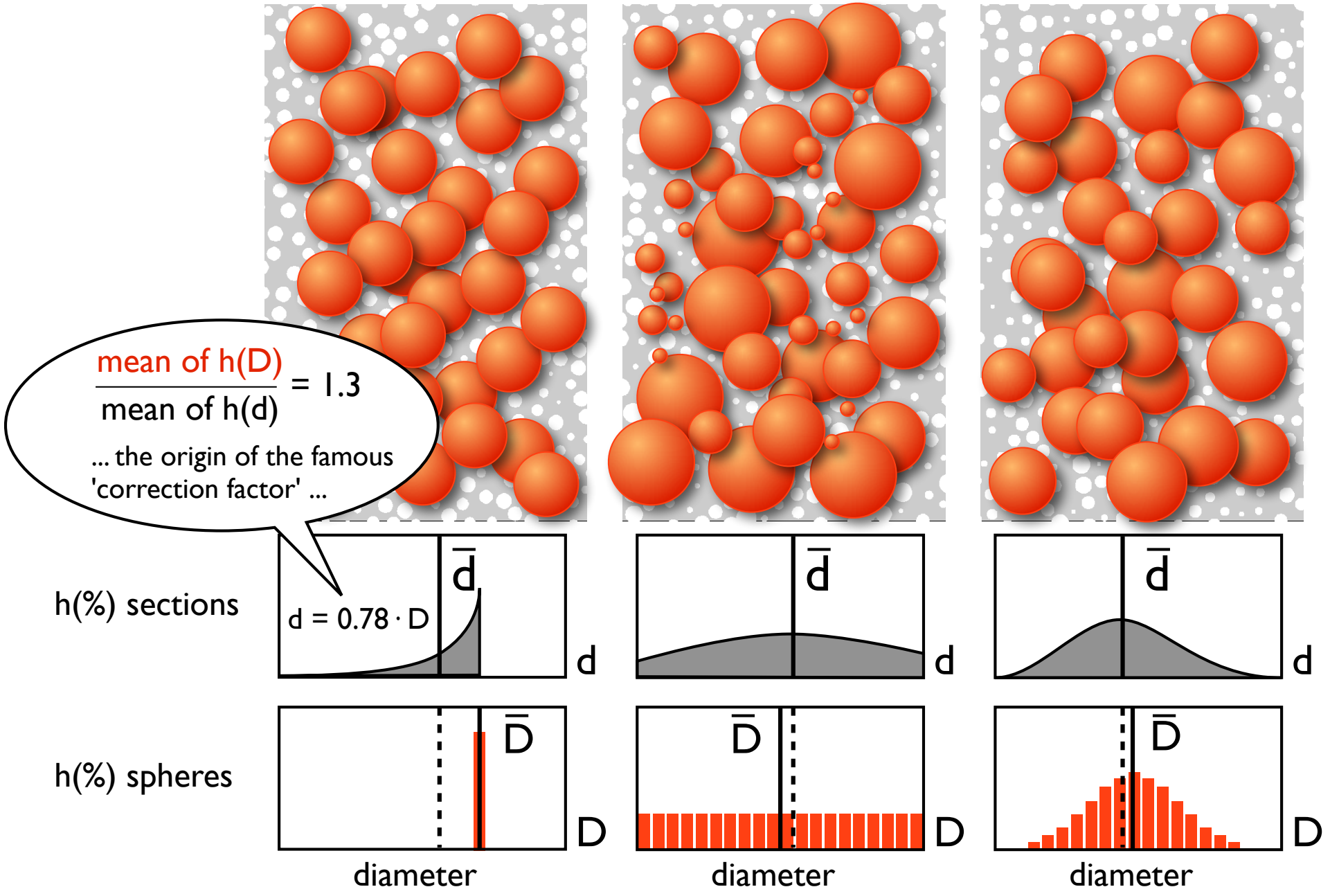




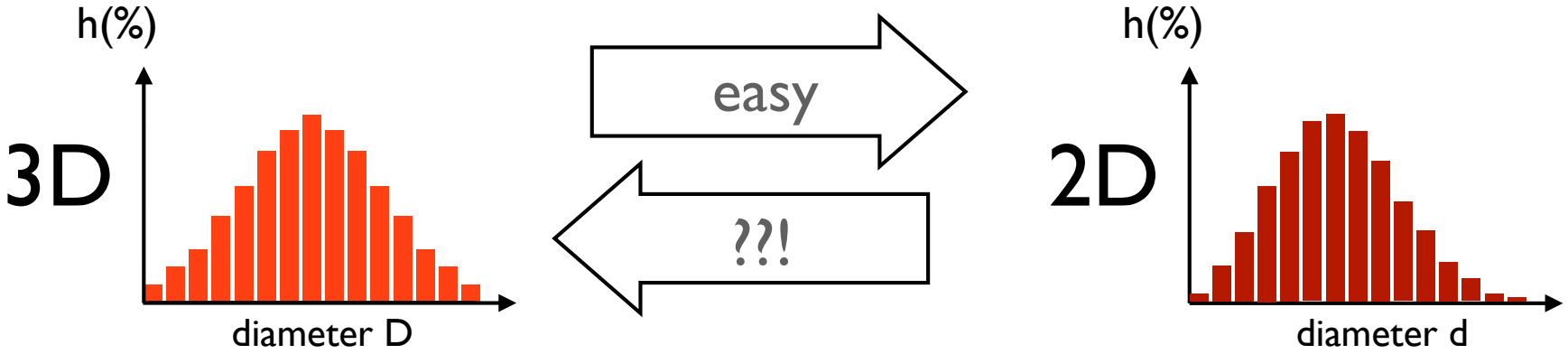




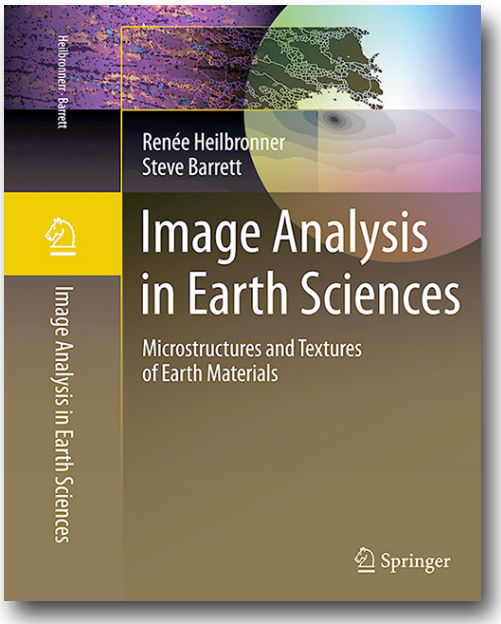
... that's why !


mean of h(D)
mean of h(d)
= 1.3
... the origin of the famous 'correction factor' ...
d = 0.78 · D
h(%) sections
h(%) spheres
d
D
diameter
diameter
diameter

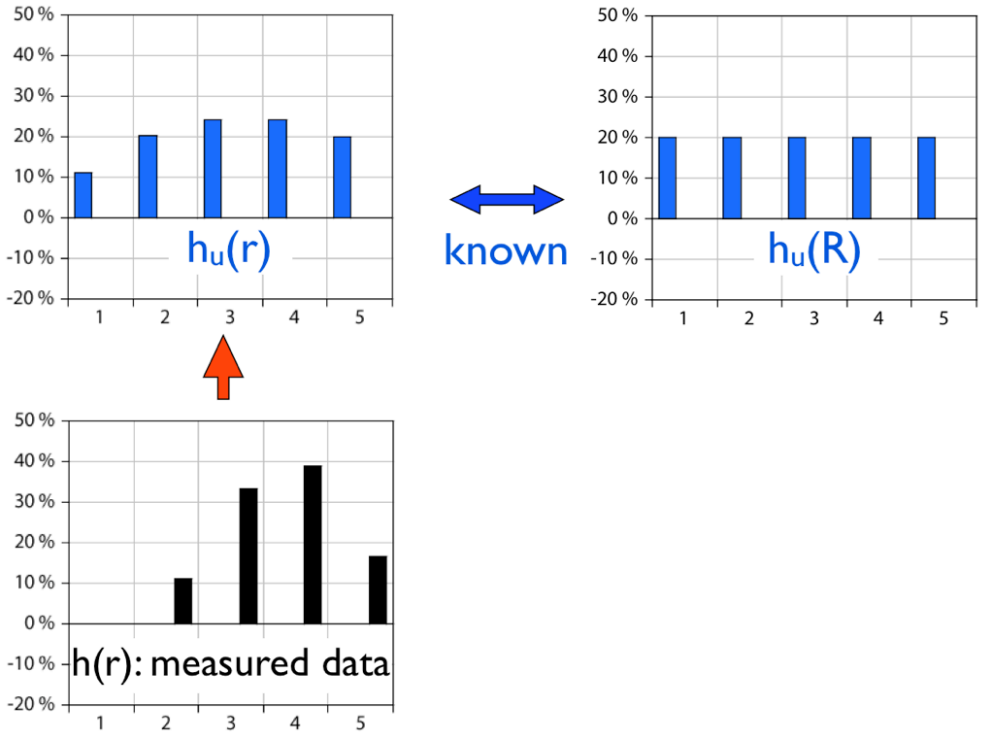
the tomato salad problem



basic idea



Heilbronner, R. and Barrett, S. (2014) Image Analysis in Earth Sciences Microstructures and Textures of Earth Materials. Spinger Verlag, Heidelberg



Basic idea behind the STRIPSTAR program:

For any uniform size distribution of spheres, $h_u(R)$, the size distribution of sections, $h_u(r)$ can be calculated.

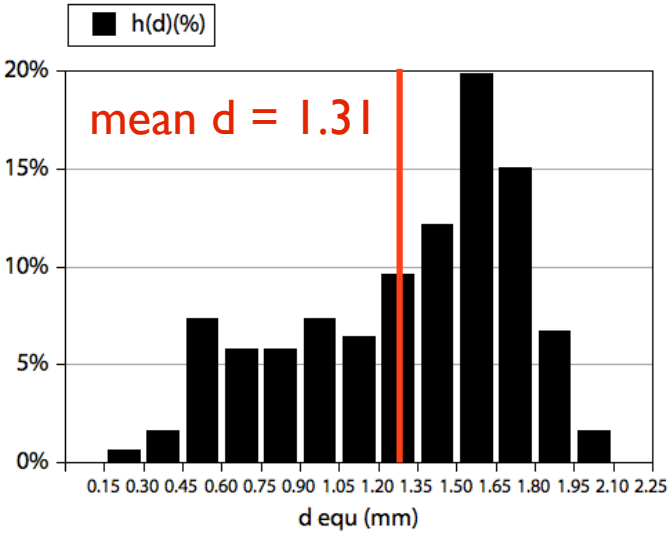
By comparing a measured size distribution, h(r), with $h_u(r)$, the parent distribution of spheres, h(R), can be derived.

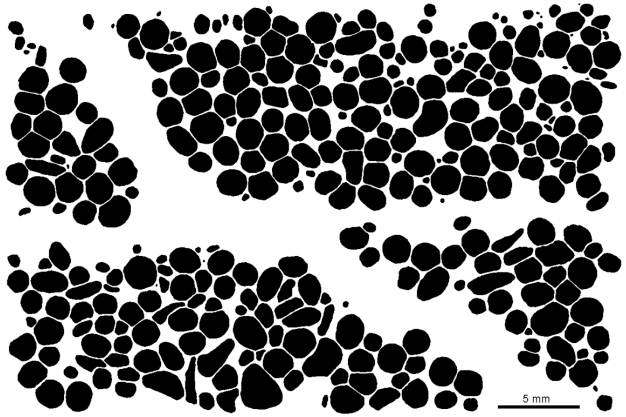
applications

- sedimentary rocks
- crystalline rocks

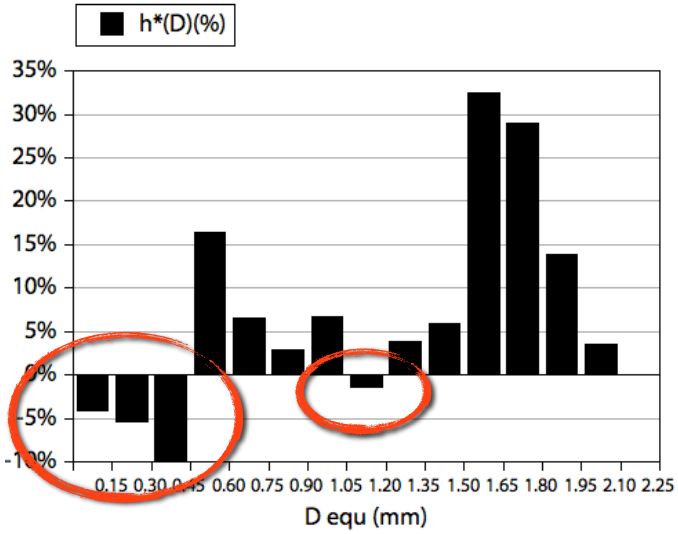
let's do it !

from 2D to 3D - from h(d) to h(D)





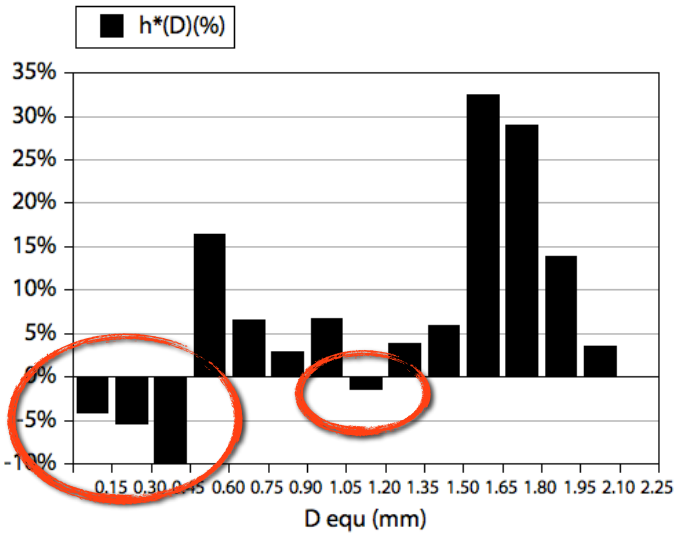
number weighted

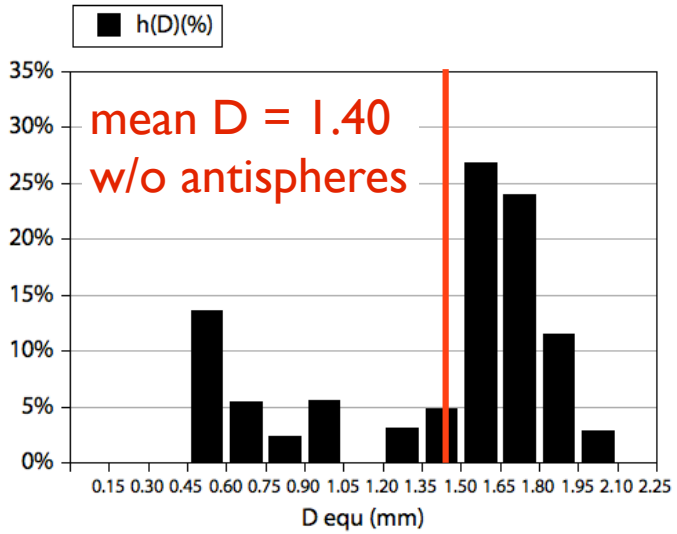


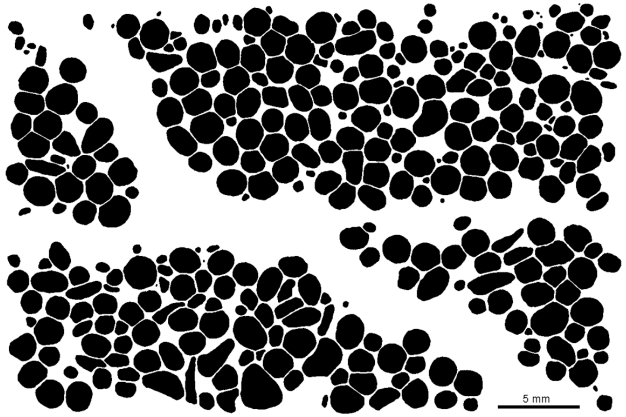
up to 10%
antispheres ?!

did it work ?

from numbers to volumes - from h(D) to vol%(D)

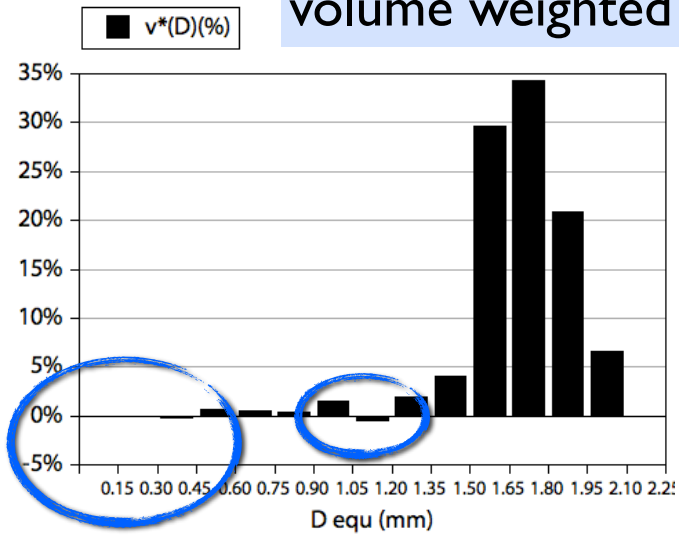


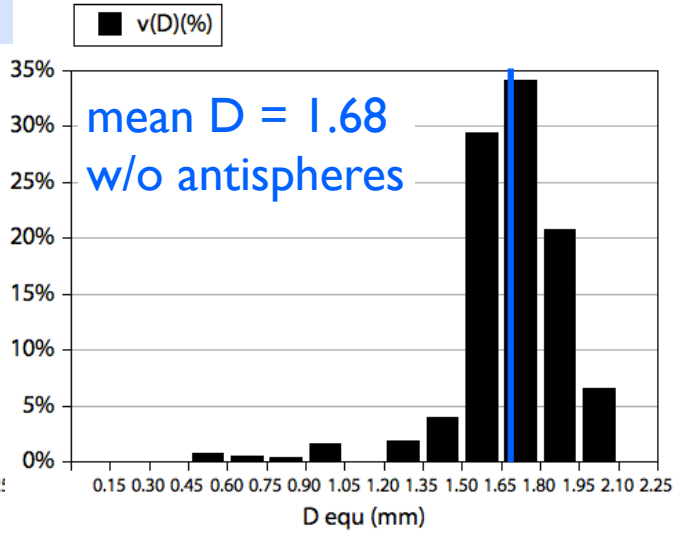




volumetrically
< 1% antispheres

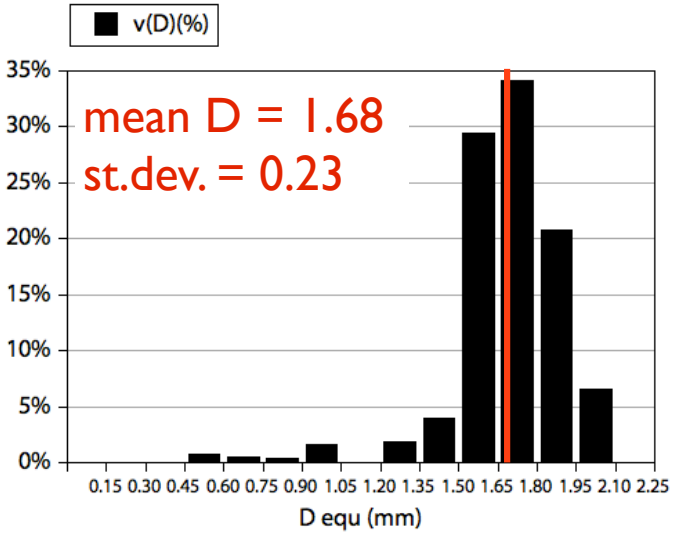
volume weighted

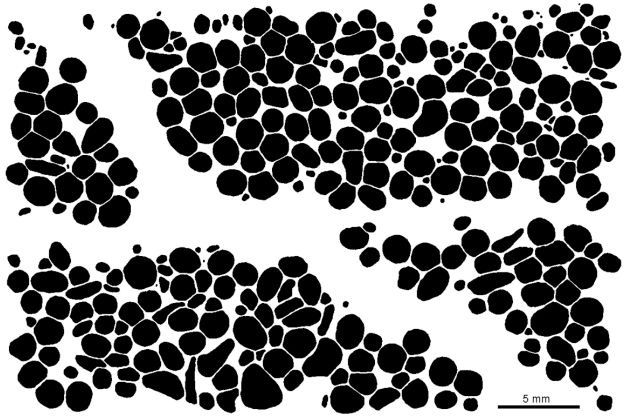




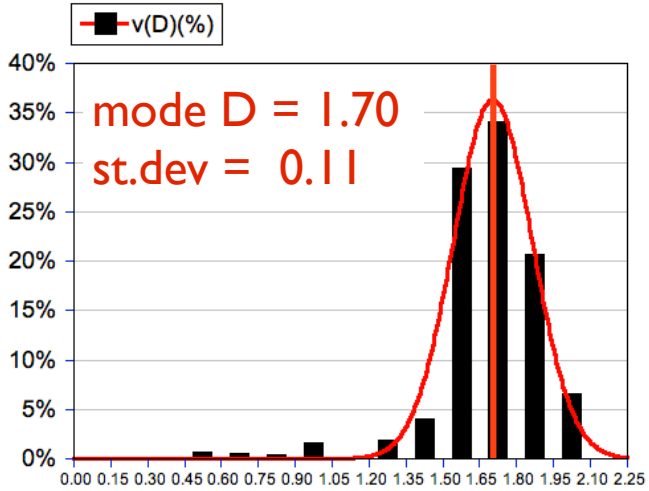
derive physically relevant grain size

from means to modes - from h(D) to vol%(D)

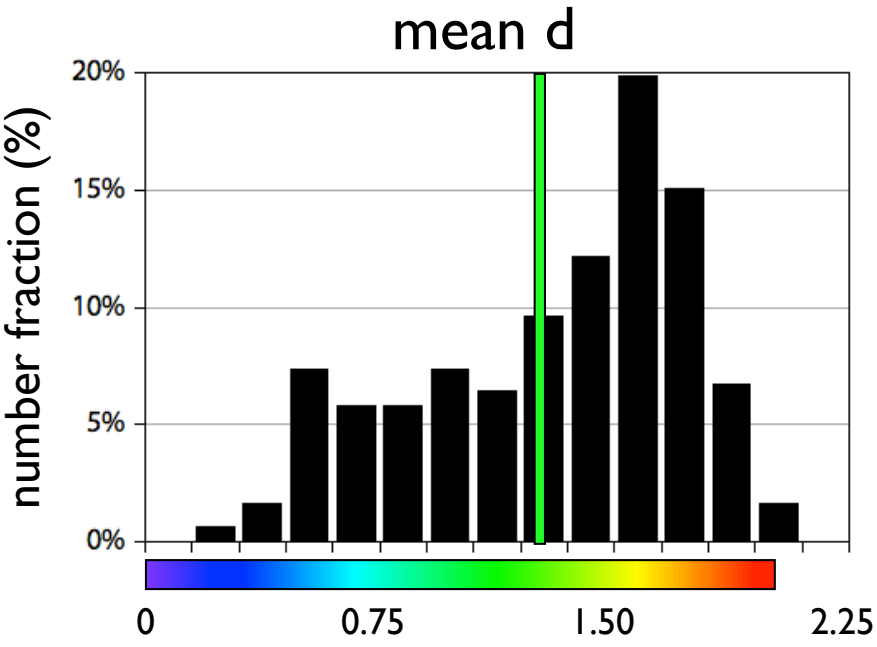


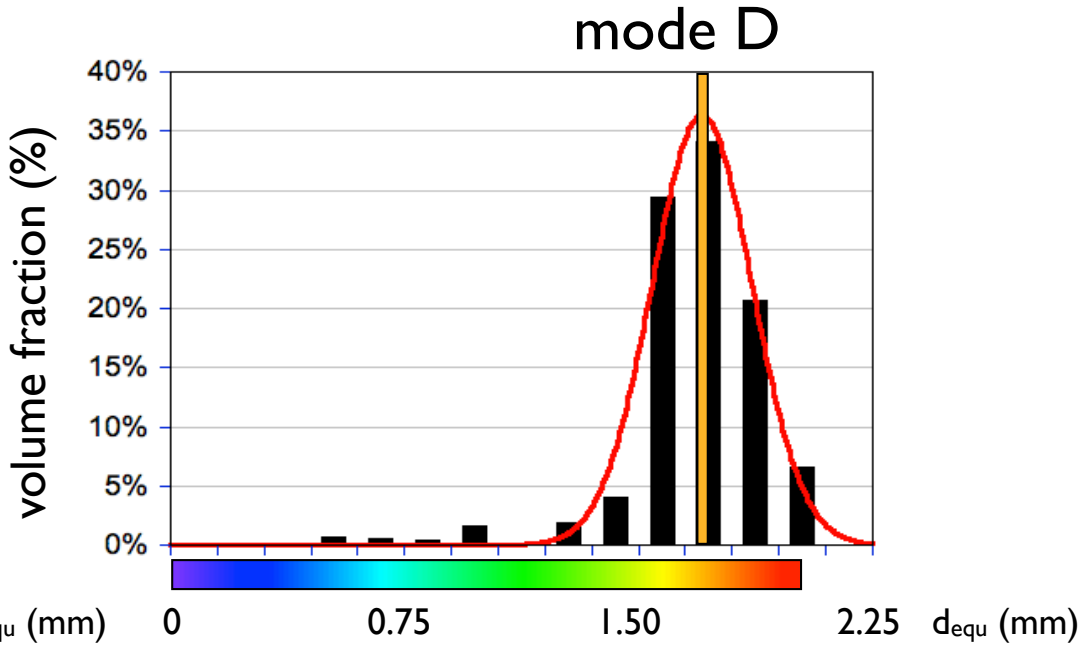


normal curve fit:
mean = mode



physically relevant grain size

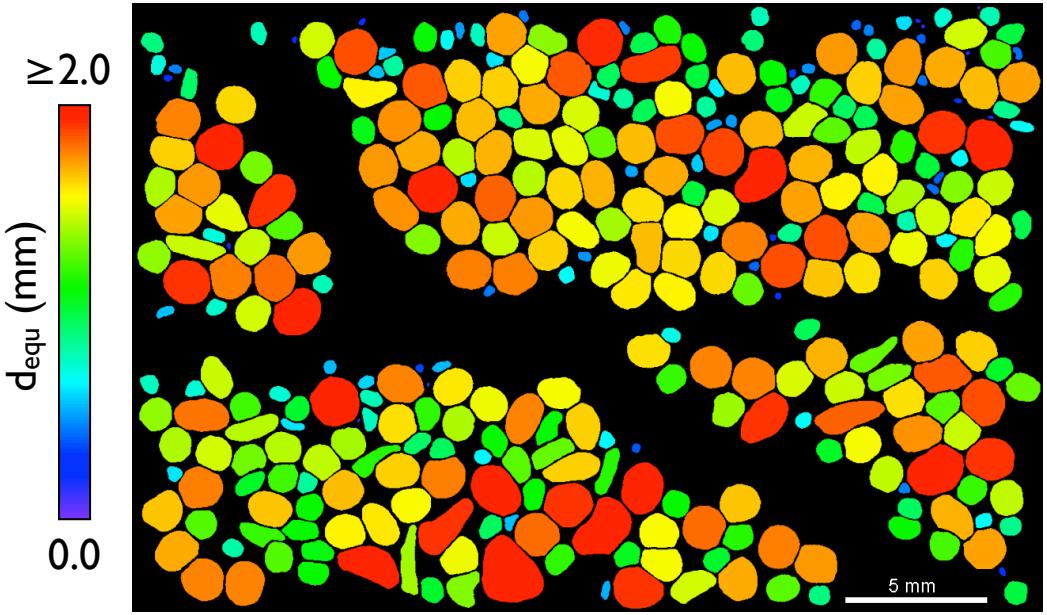




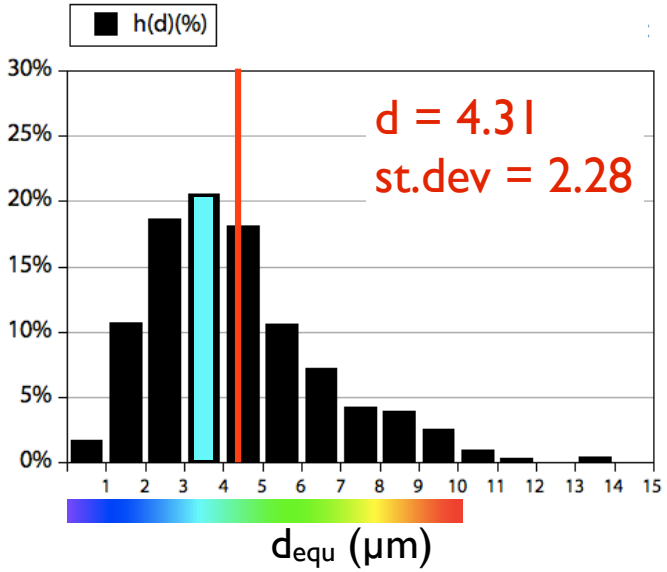
number-weighted
2D mean 1.31 ≠ *visual impression*
2D st.dev. 0.43

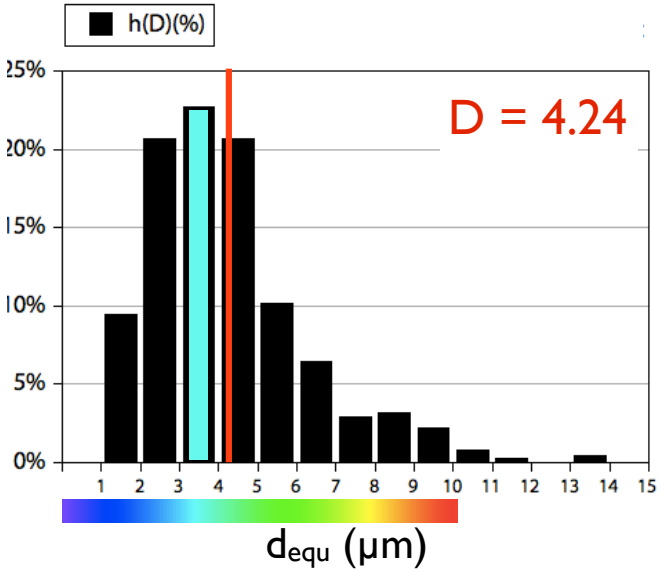
volume-weighted
3D mode 1.70 = *visual impression*
3D st.dev. 0.11

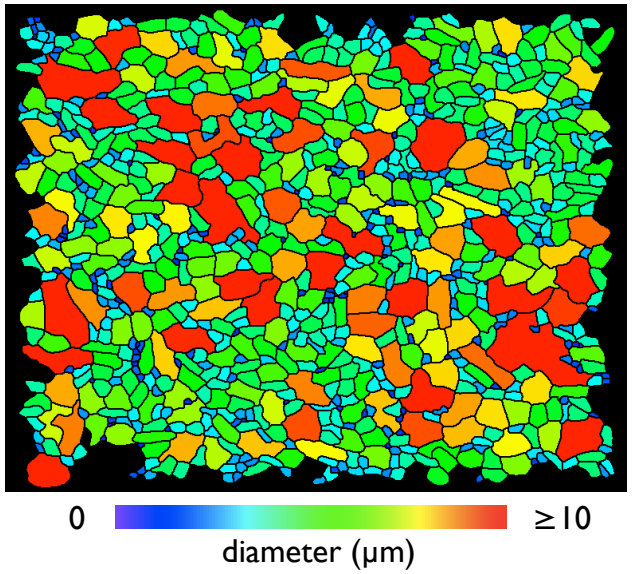
note sorting !

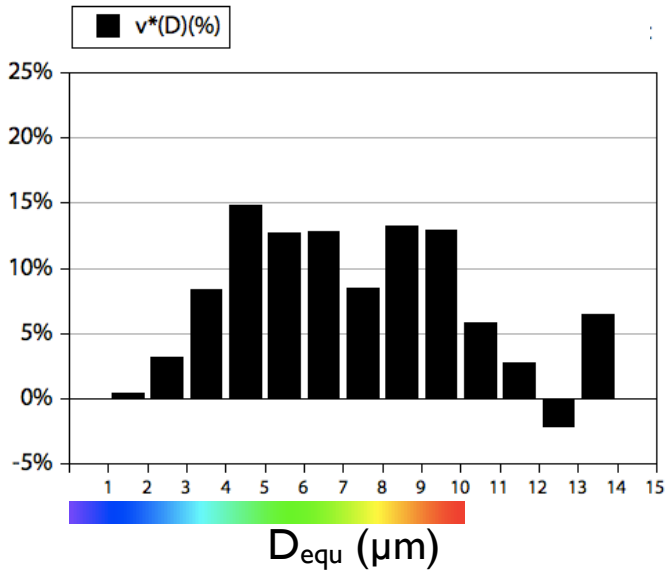


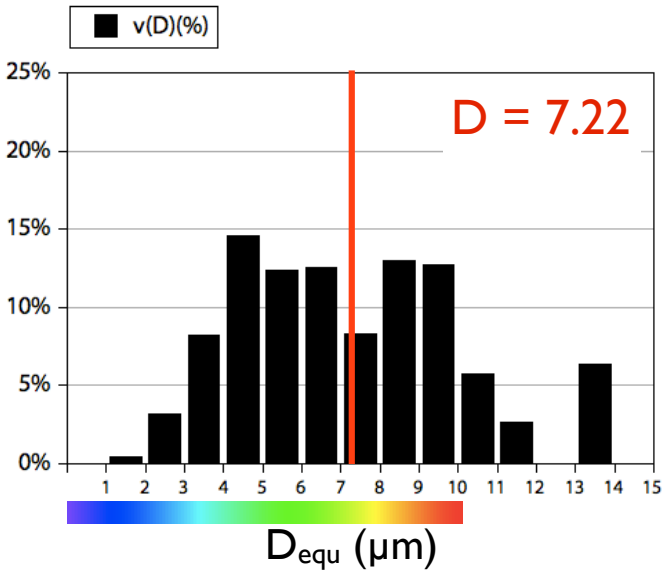
another example

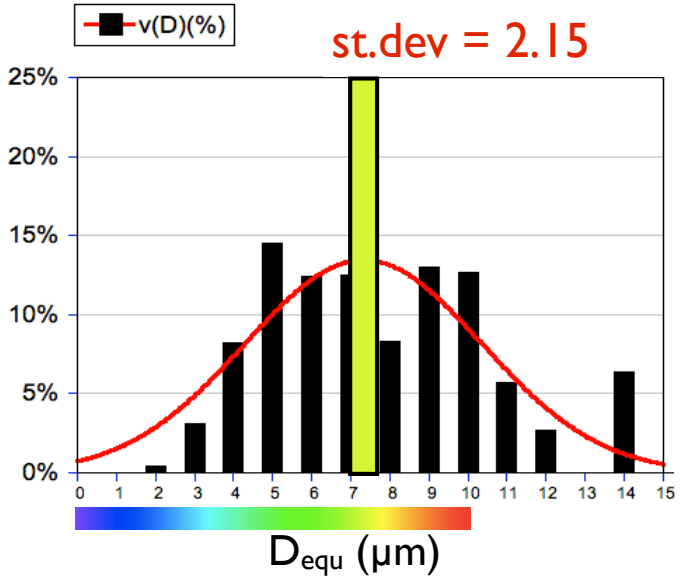






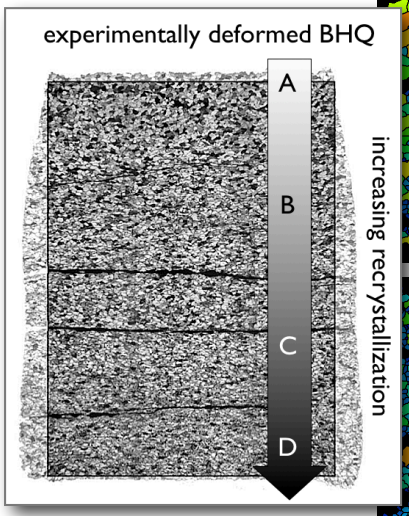


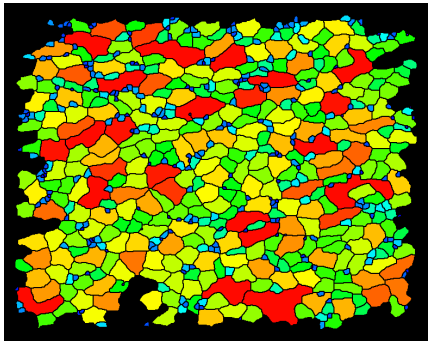


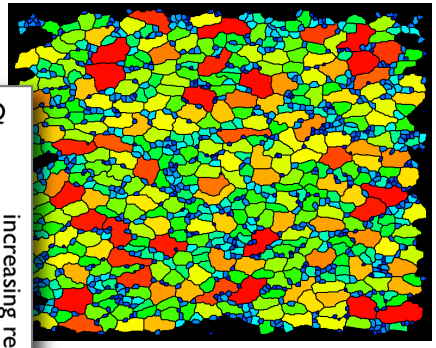


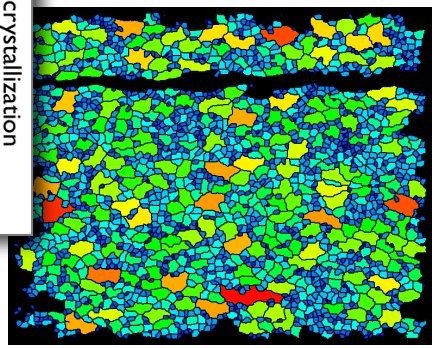
... and another one ...

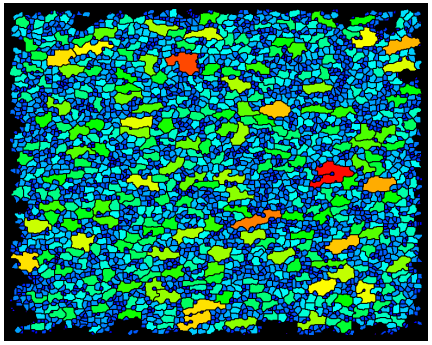
mode1 mode2



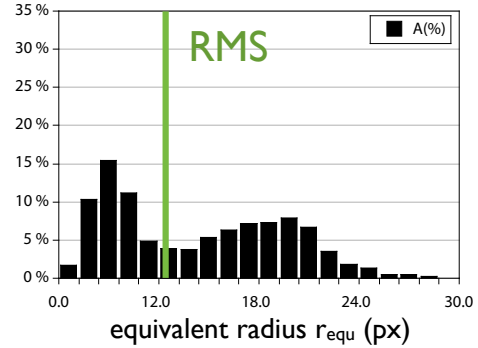


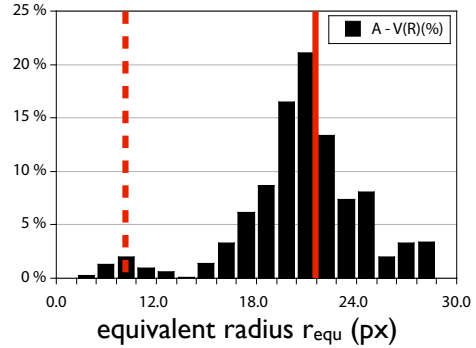




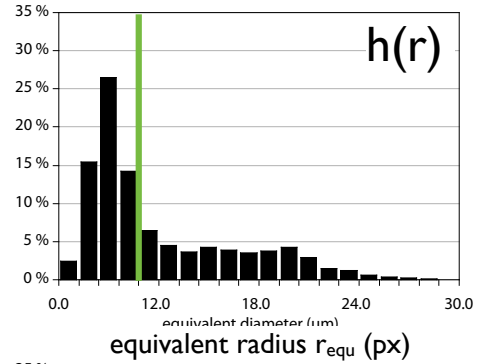


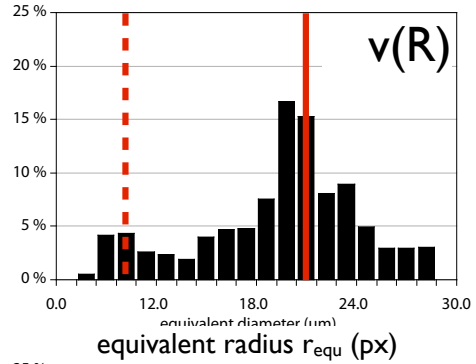
12.4



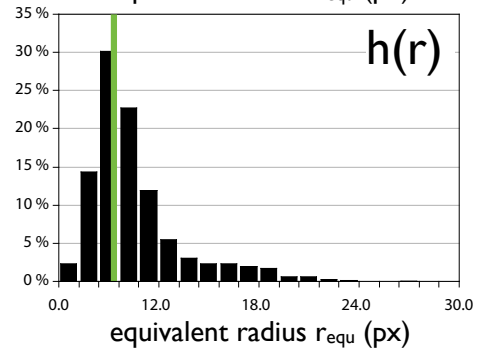


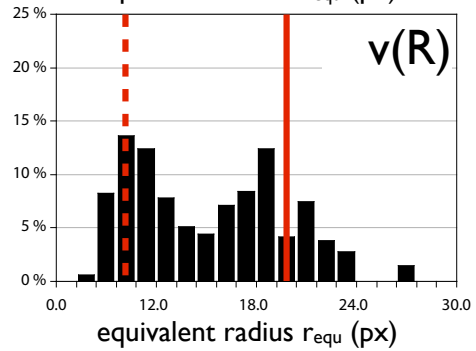
9.5



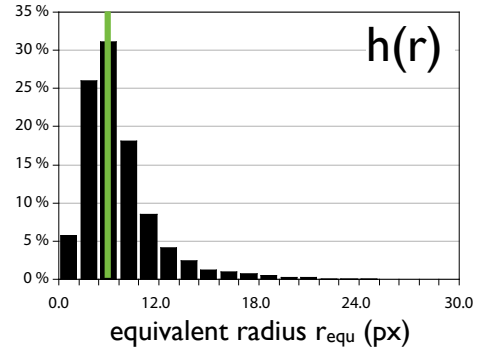


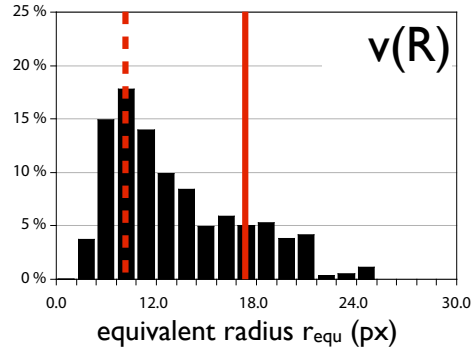
6.7





5.2

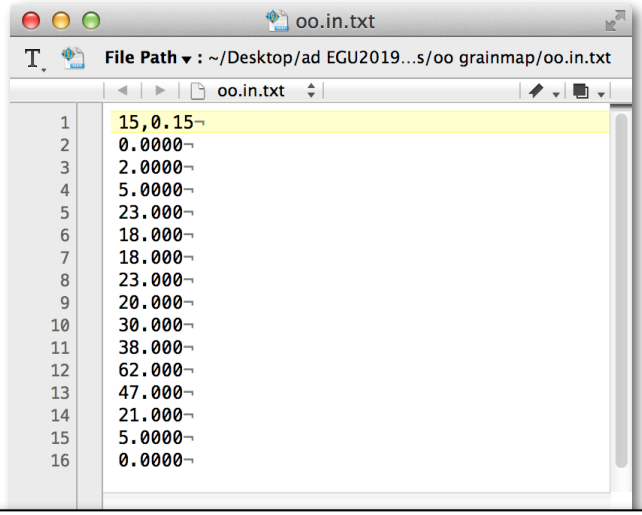


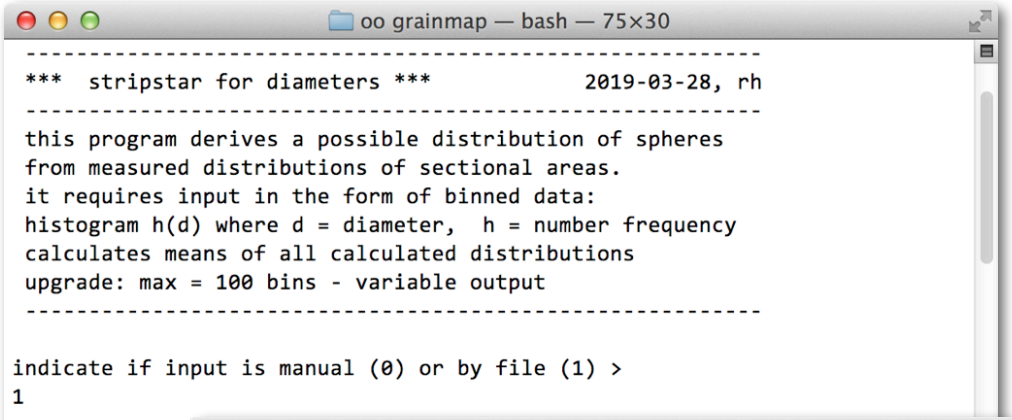


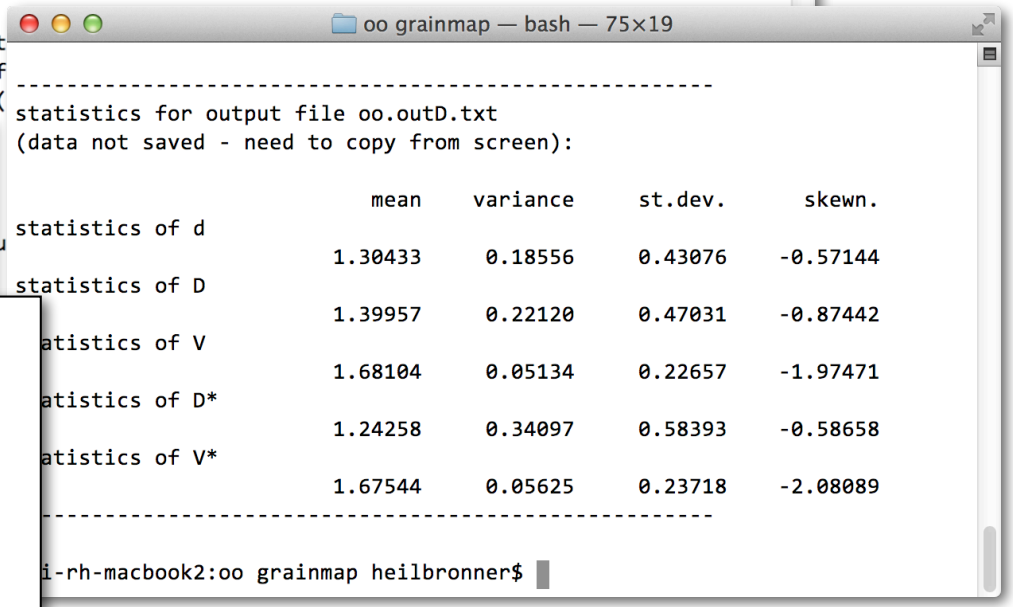
software and short cuts

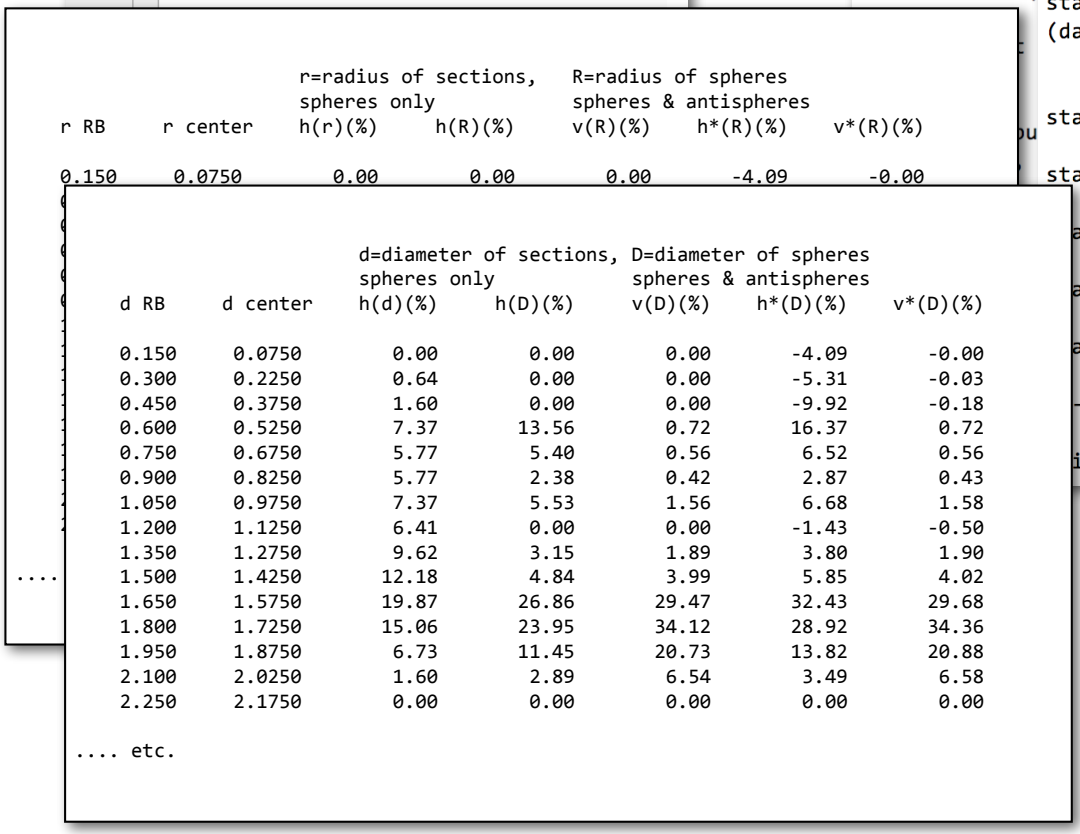
- stripstar

stripstar (good old Fortran)



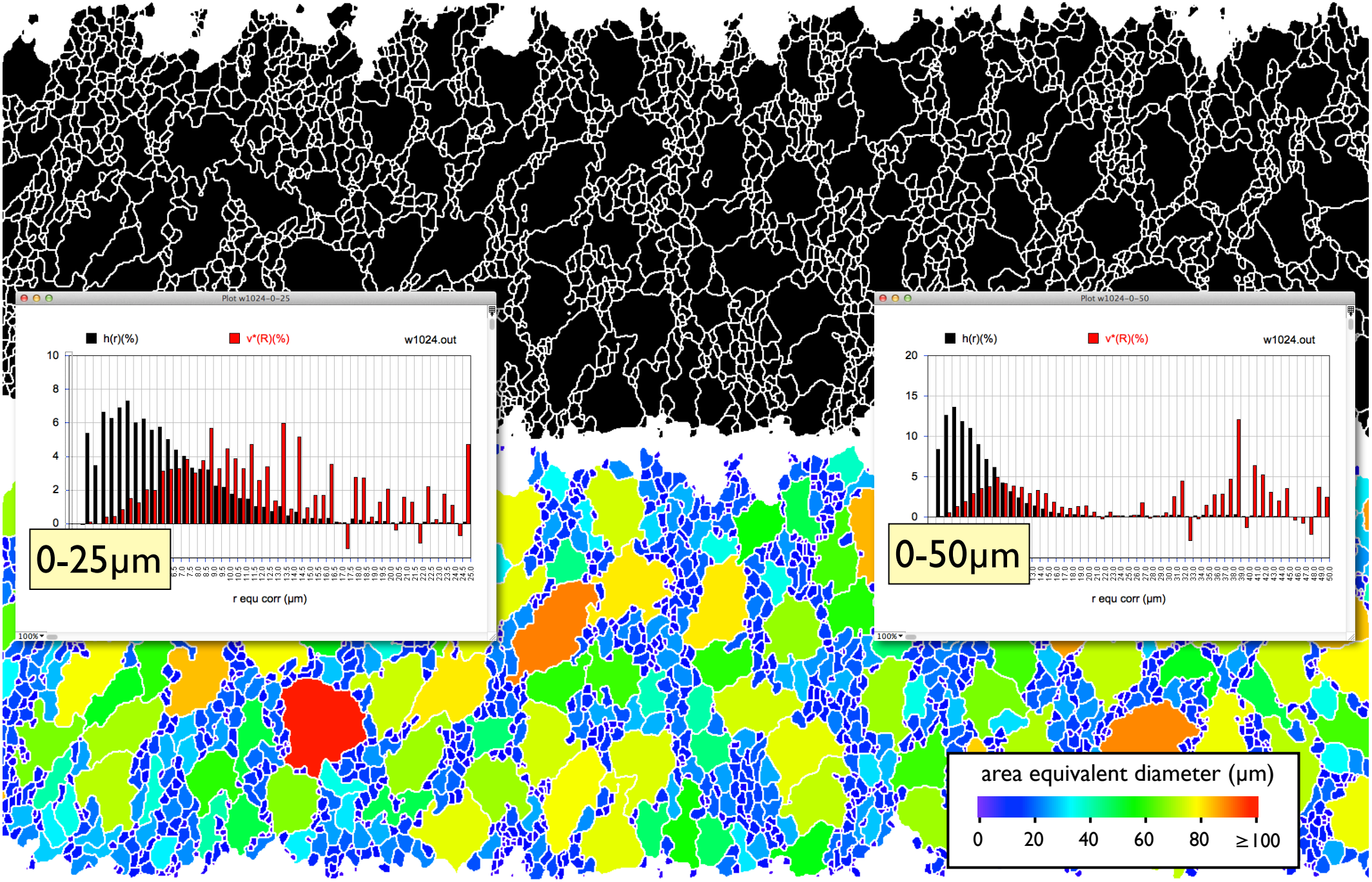






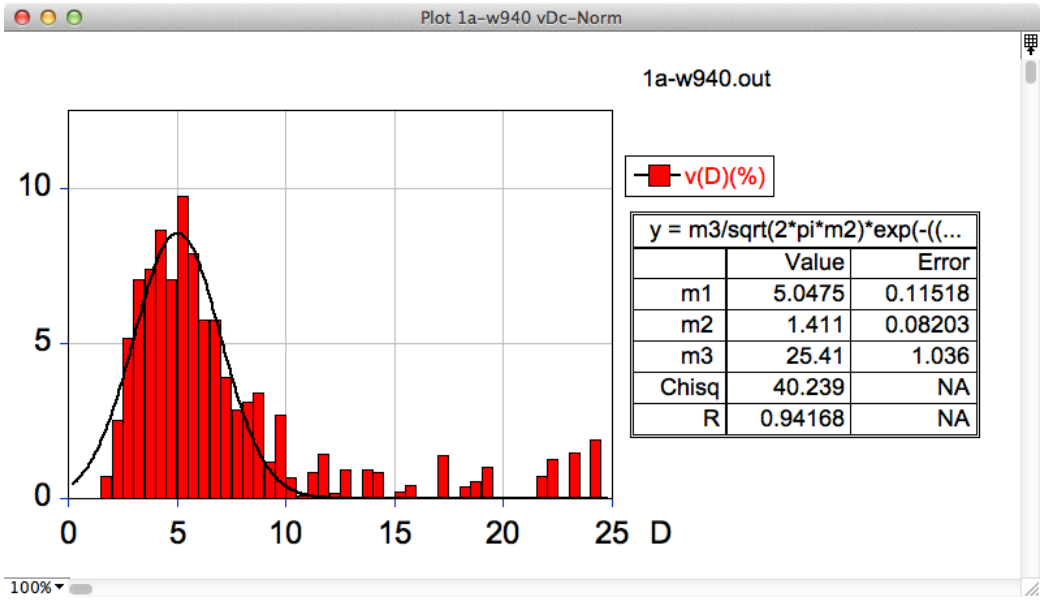
stripstar2019-R.f
stripstar2019-D.f

spread sheet (Kaleidagraph)

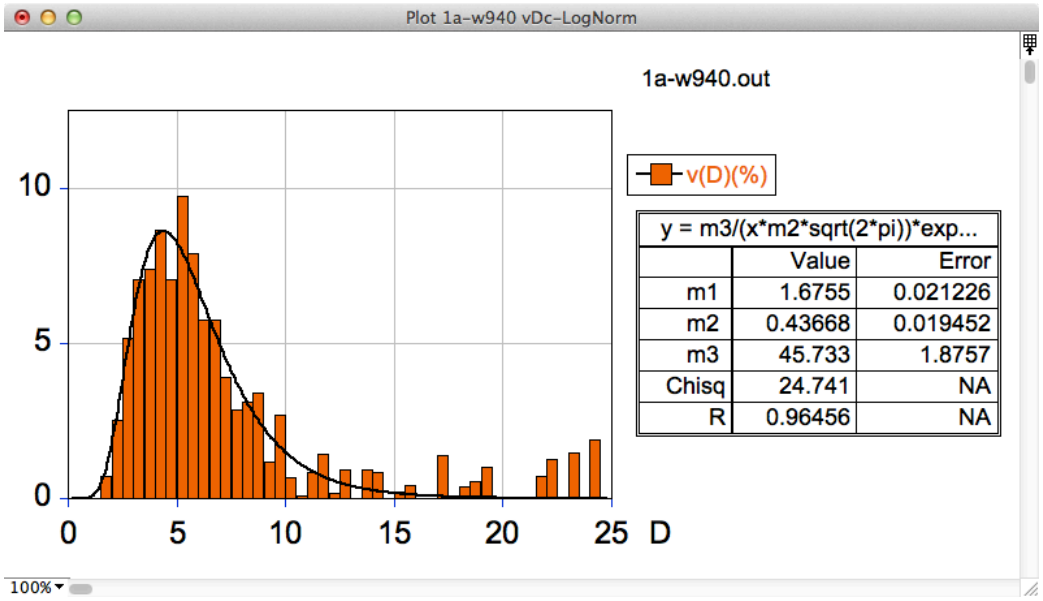


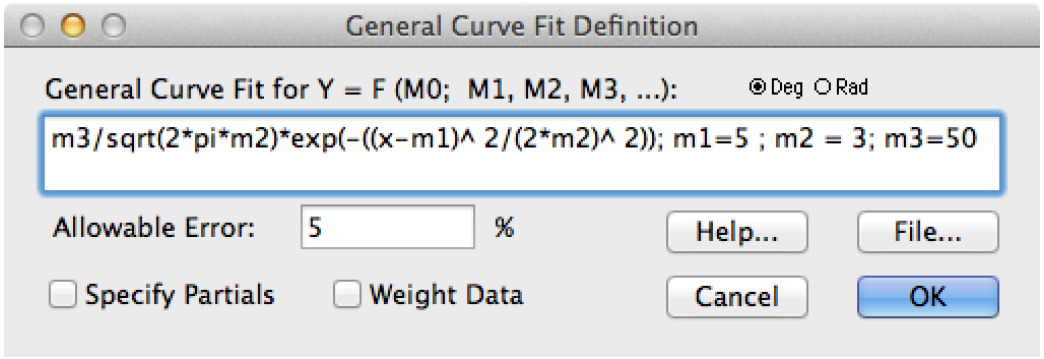
how to get a mode

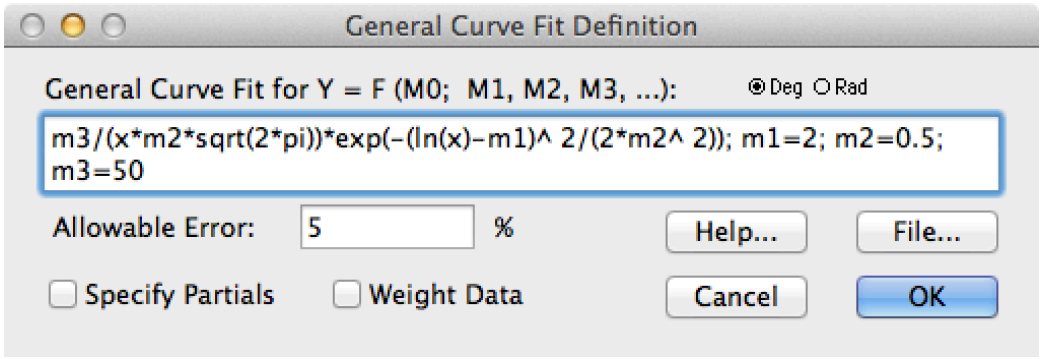
normal curve fit



lognormal curve fit

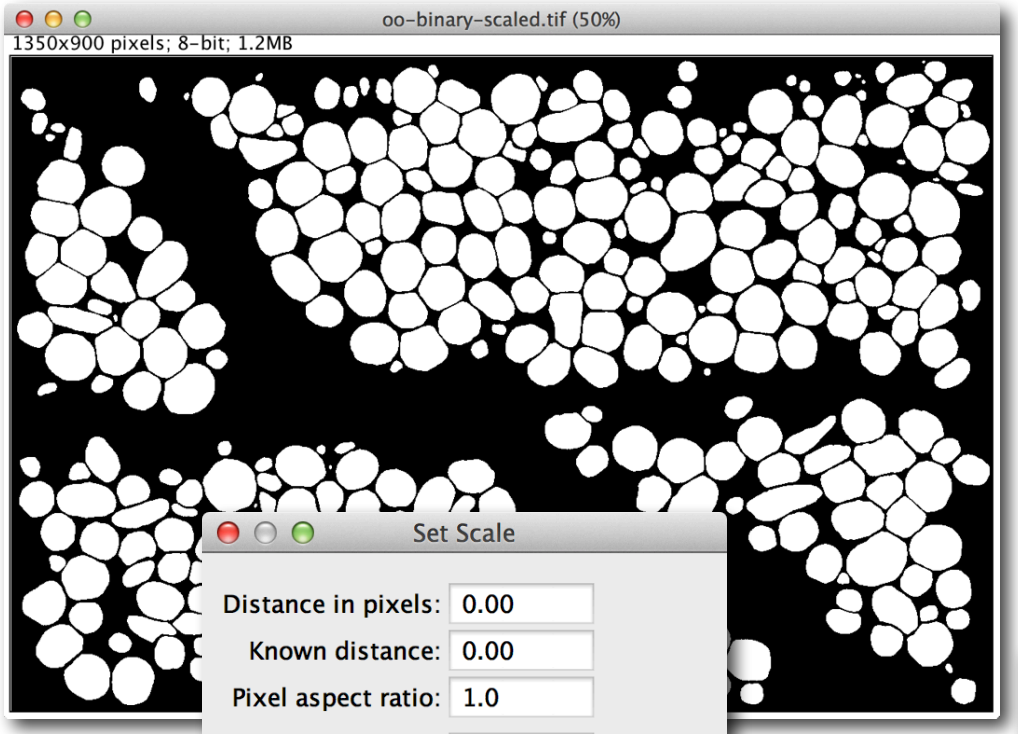


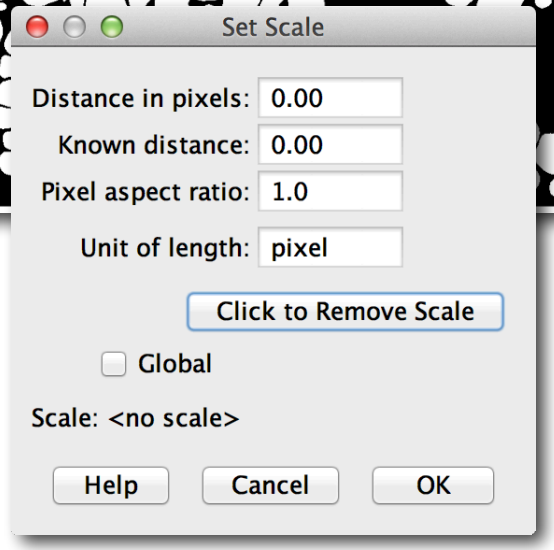


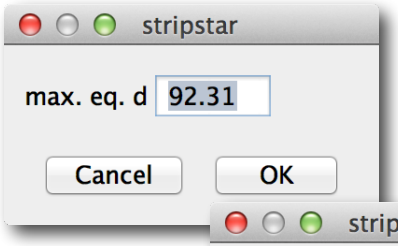


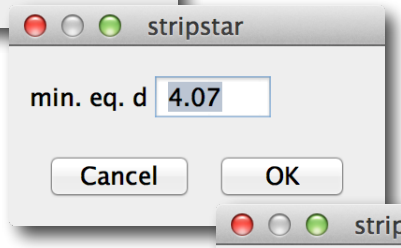
the stripper

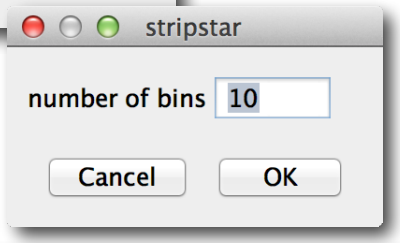
(ImageJ macro by Rüdiger Kilian)

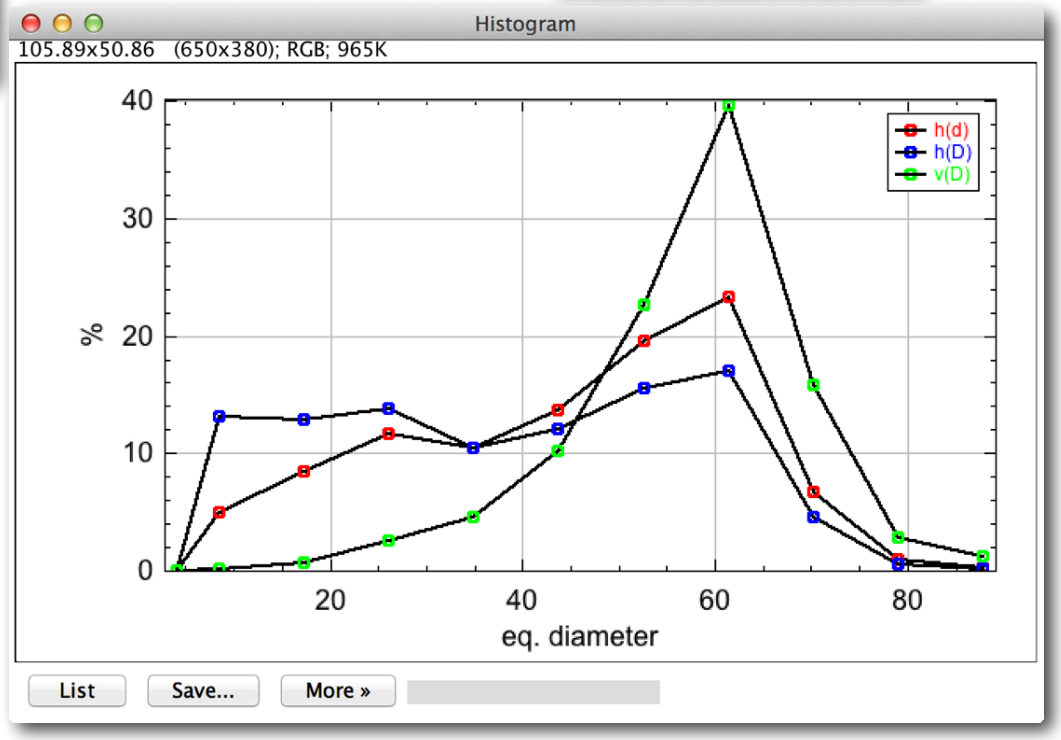






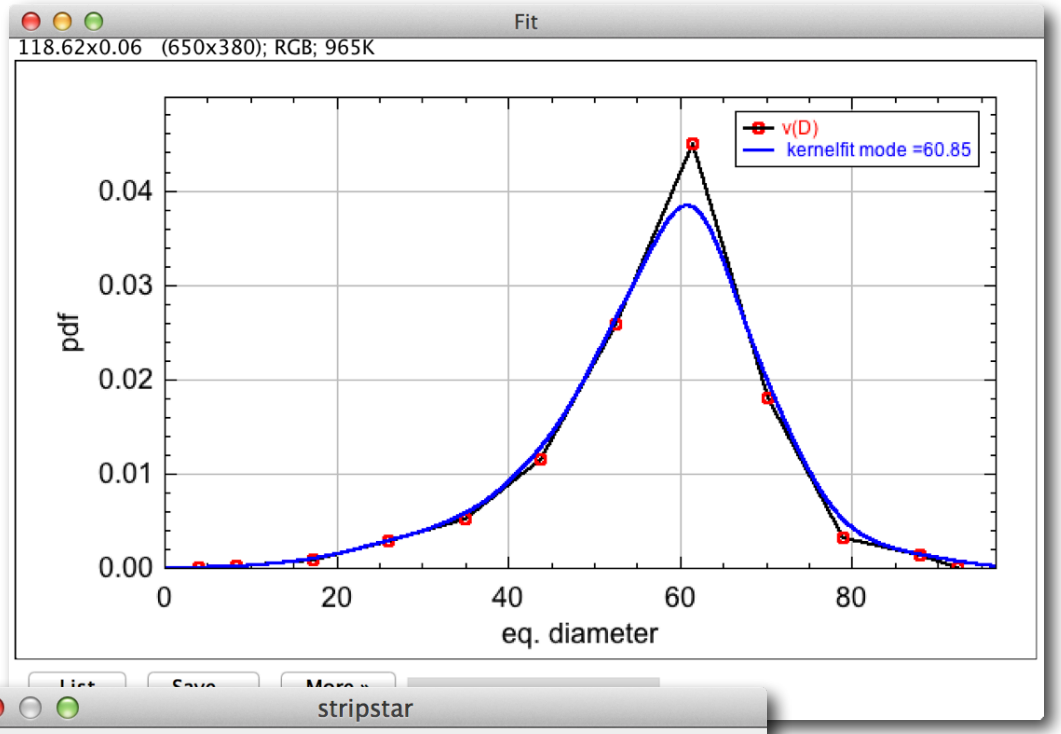


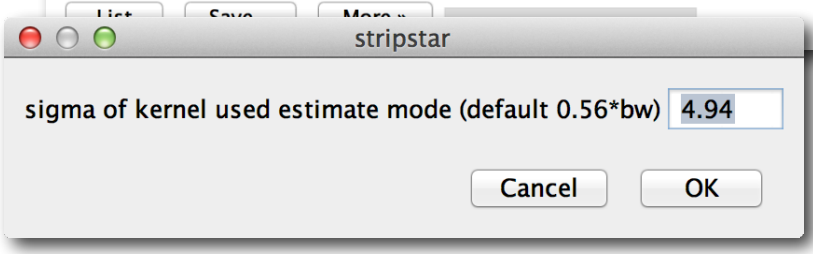




scale image
check if background = black
check non-invertig LUT

the stripper





Results

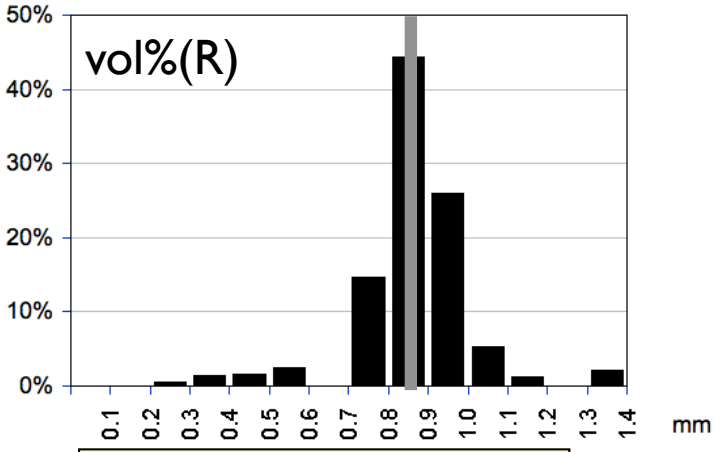
	Area	Perim.	EqDia	bin	bincenter	h(d)	h(D)	v(D)	h(D)*
1	608	92.083	27.823	4.068	8.480	4.956	13.058	0.088	0.088
2	13	11.899	4.068	12.892	17.304	8.455	12.861	0.696	0.696
3	644	94.326	28.635	21.716	26.128	11.662	13.740	2.511	2.511
4	556	87.154	26.607	30.540	34.952	10.496	10.451	4.527	4.527
5	190	51.941	15.554	39.364	43.776	13.703	12.034	10.180	10.180
6	2899	201.480	60.755	48.188	52.599	19.534	15.517	22.682	22.682
7	1362	137.296	41.643	57.011	61.423	23.324	17.043	39.561	39.561
8	377	73.841	21.909	65.835	70.247	6.706	4.561	15.805	15.805
9	82	33.799	10.218	74.659	79.071	0.875	0.559	2.757	2.757
10	3669	225.622	68.348	83.483	87.895	0.292	0.176	1.193	1.193
11	38	21.556	6.956	0.000	0.000	0.000	0.000	0.000	0.000
12	2180	180.995	52.685	0.000	0.000	0.000	0.000	0.000	0.000
13	170	47.456	14.712	0.000	0.000	0.000	0.000	0.000	0.000
14	210	56.426	16.352	0.000	0.000	0.000	0.000	0.000	0.000

mean vol%(D) = 60.85 px

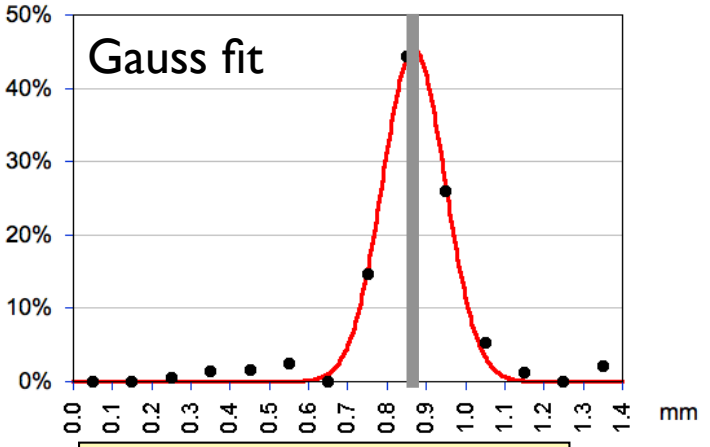
$\Longrightarrow$ mean D = 1.76 mm

$\Longrightarrow$ mean R = 0.88 mm

comparison:



→ R = 0.86 ± 0.15 mm



→ R = 0.87 ± 0.11 mm

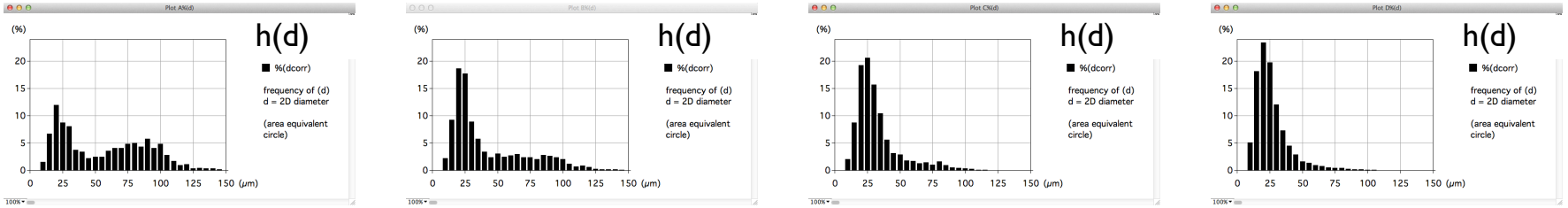
stripstar.m

stripstar.m

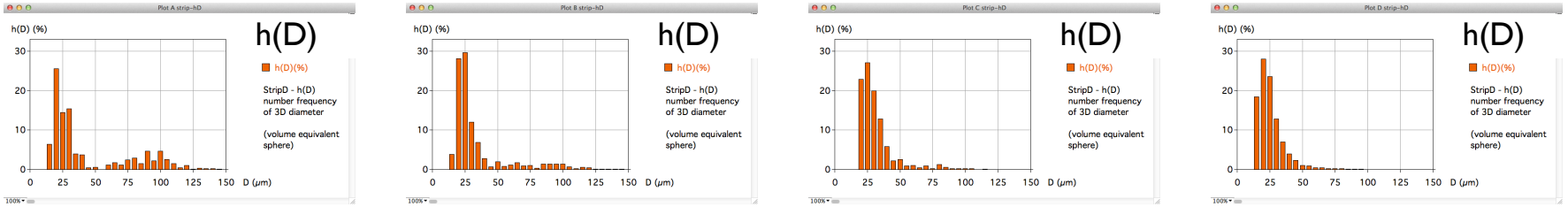
= stripstar.f

stripstar versus shortcuts

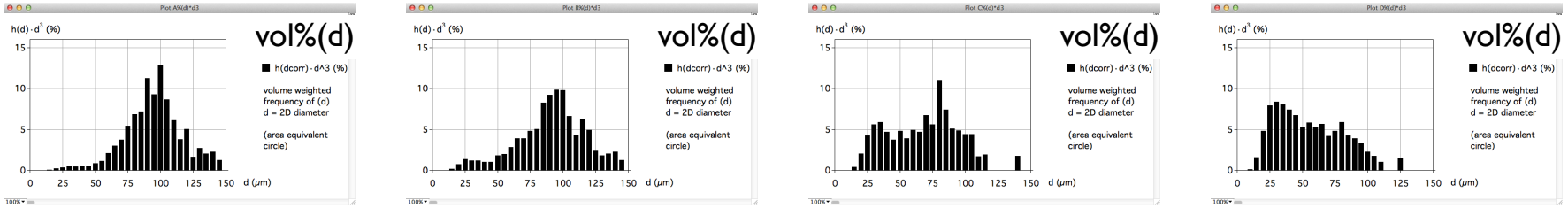
number-weighted



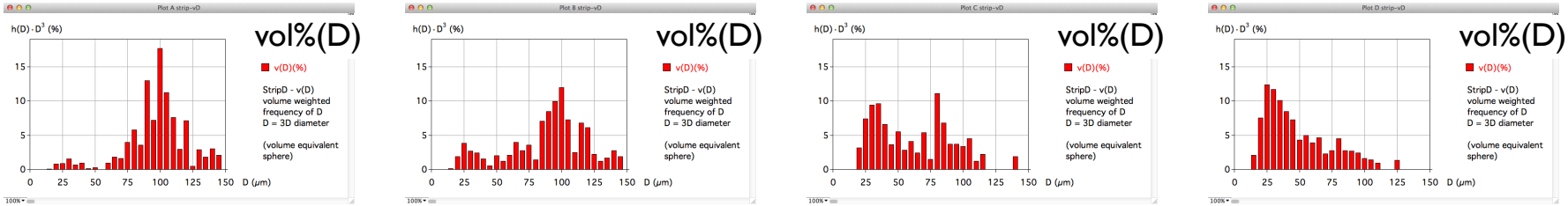
stripstar



volume-weighted



stripstar

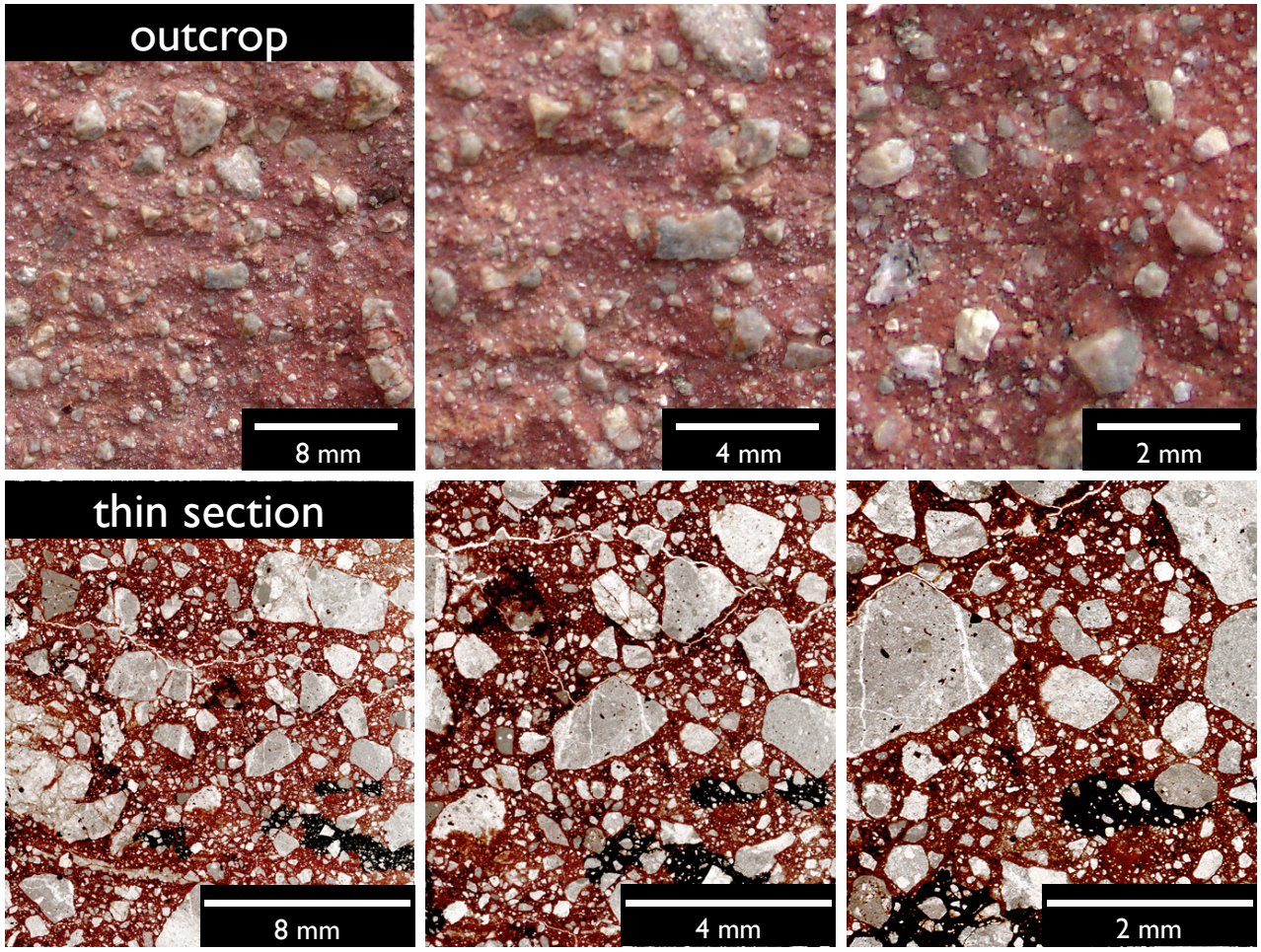


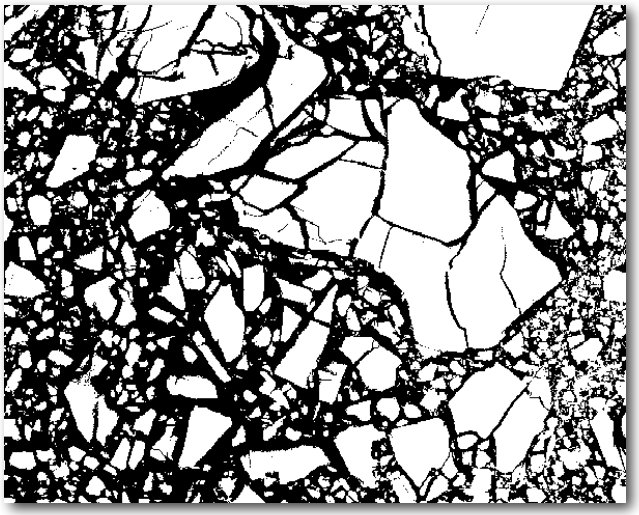
5
fractal grain size distributions

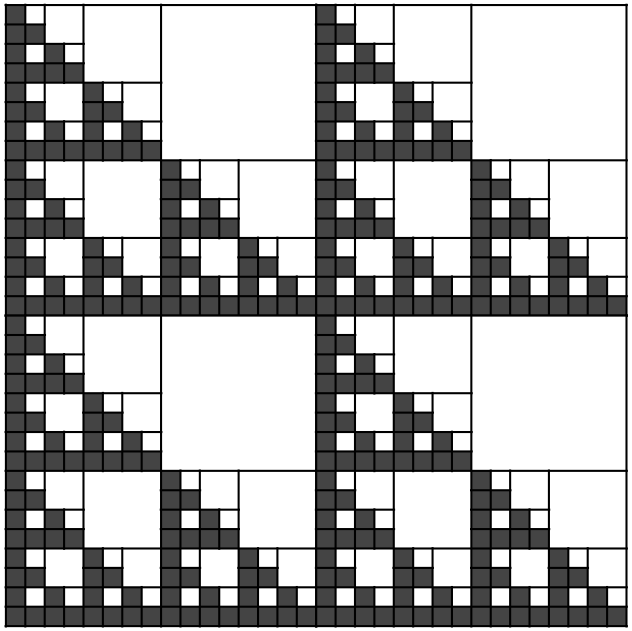
the fractal dimension

- selfsimilarity
- D_{3d} D_{2d} and E

self-similarity

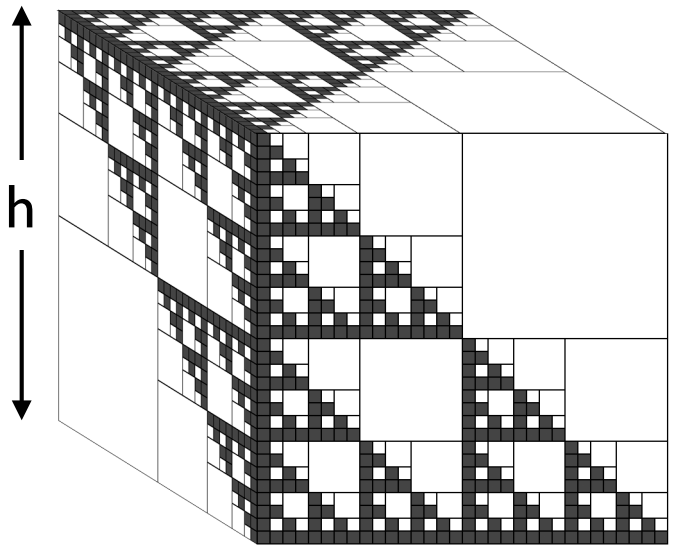






fractal distribution
→ grain size ratio

fractal fragmentation



$$D = \frac{\ln (n_2/n_1)}{\ln (r_1/r_2)}$$

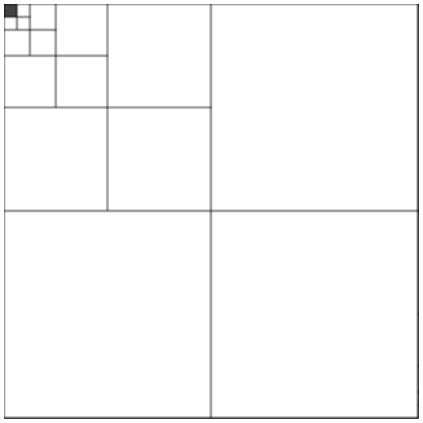
$r_1 = h/2$ $\quad n_1 = 2$
$r_2 = h/4$ $\quad n_2 = 12$

Example: F = 8 fragments
N = 2 remain unfragmented

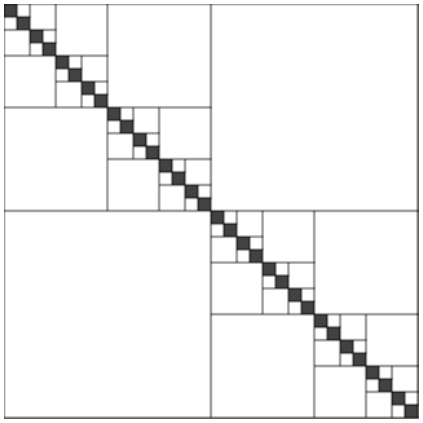
Fragmentation fraction:
f = (F-N) / F = 6 / 8

$$D = \frac{\ln (n_2/n_1)}{\ln (r_1/r_2)} = \frac{\ln (6)}{\ln (2)} = 2.585$$

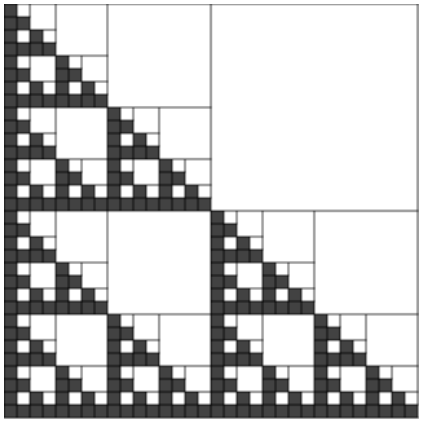
Map views:



1/8 fragmented



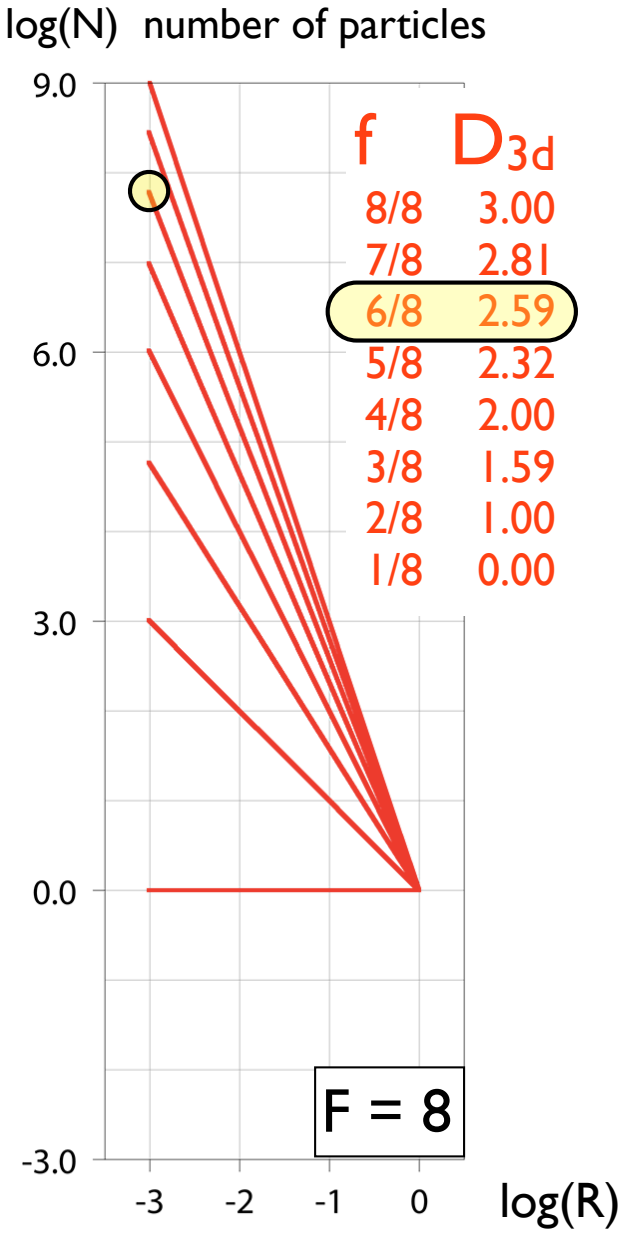
4/8 fragmented



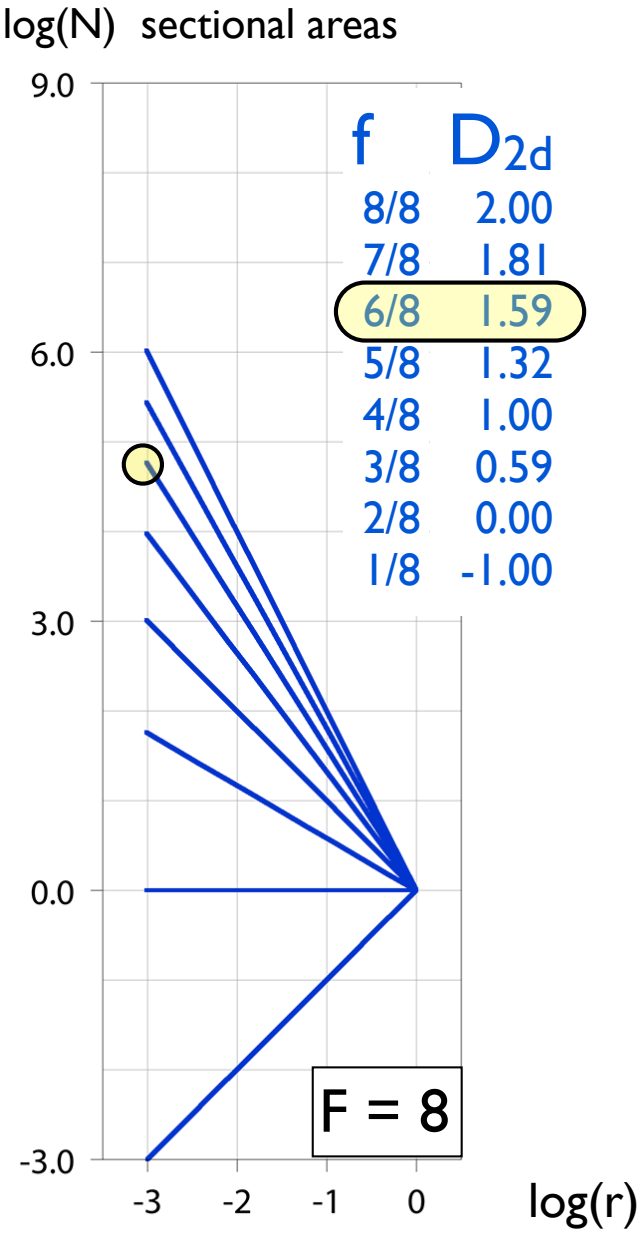
6/8 fragmented

fractal dimensions

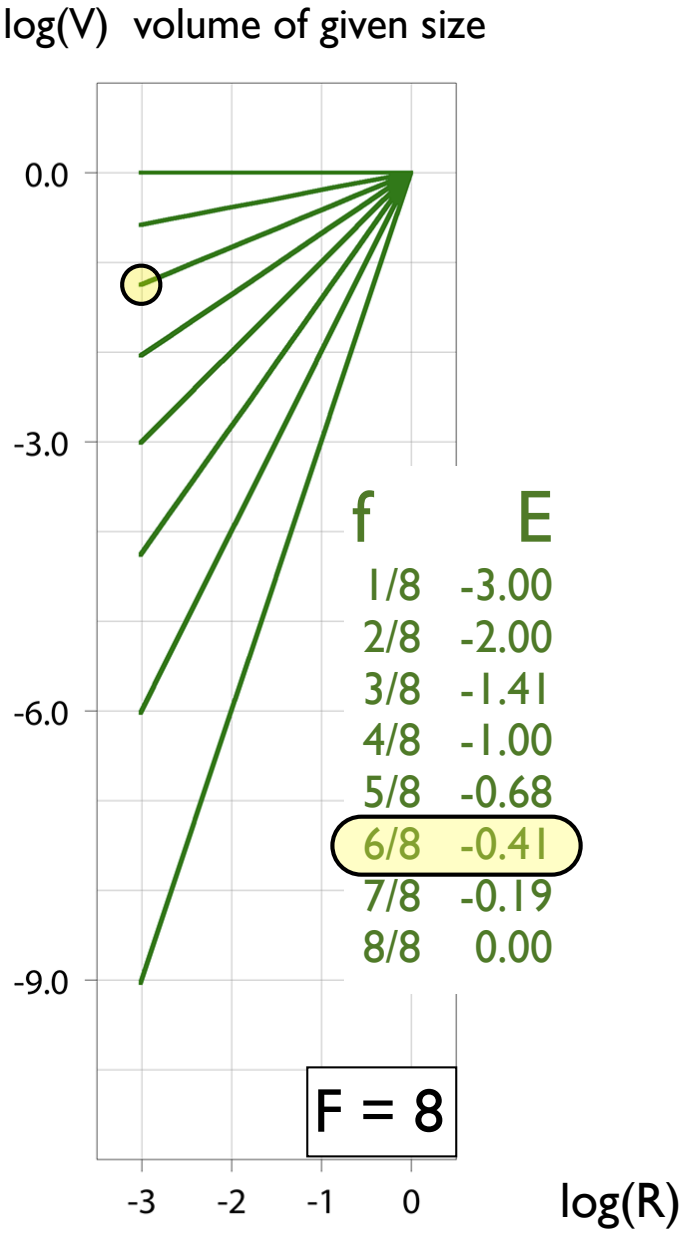
D_{3d}



D_{2d}



E

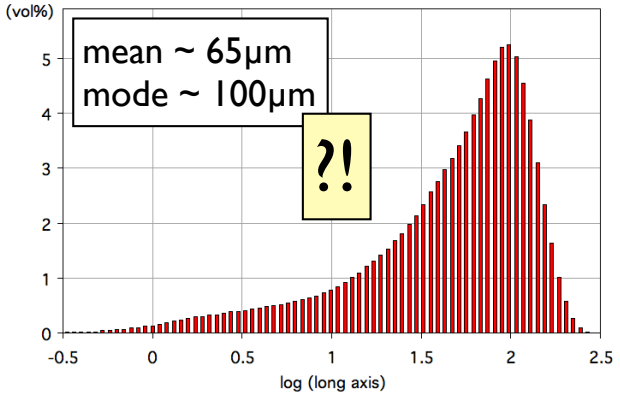


how to determine D

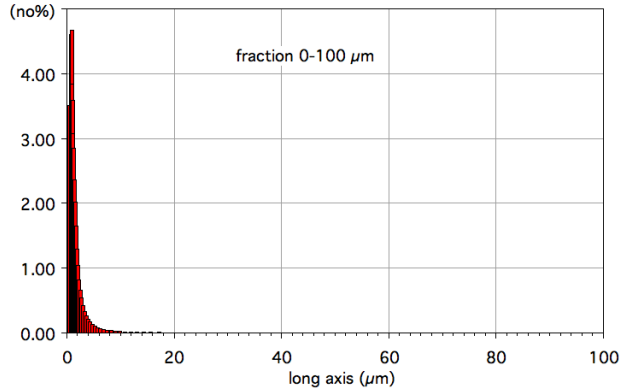
- particle analyzer
- thin sections

fractal dimension from sieved data

1

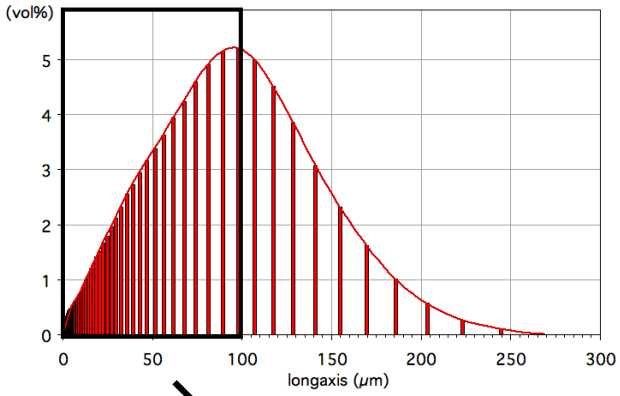


4

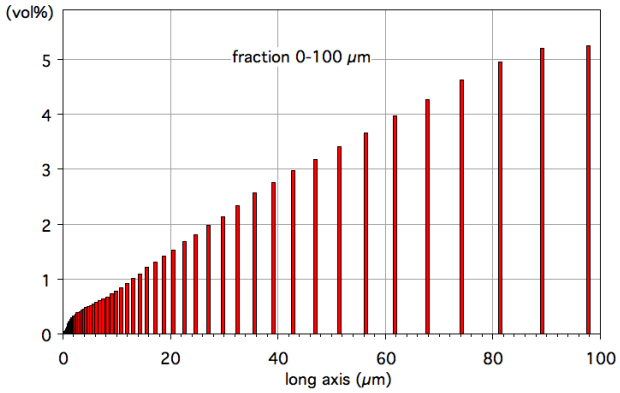


for sieved data
$D_{3d} \leq 3$

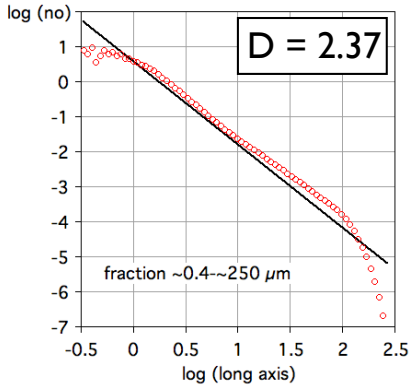
2



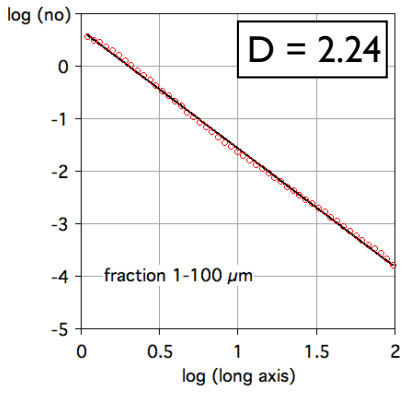
3



5



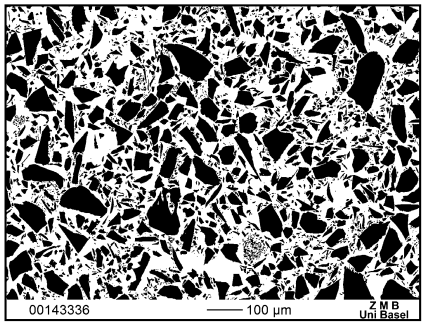
6

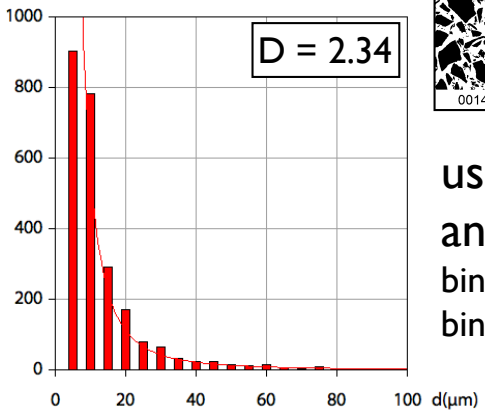


1. Clean original data:
   Plot: vol% vs. log(LA)
2. Linear LA: vol% vs. LA(µm)
   Plot: full range
3. Linear LA: vol% vs. LA(µm)
   Plot: (1µm ≤ d ≤ 100µm)
4. Convert vol% → no%:
   vol% / LA^3 = no%
   Plot: no% vs LA(µm)
5. Convert no→ log(no), LA→ log(LA)
   Plot: log(no) vs. log(LA)
   Plot: full range
6. Plot: log(no) vs. log(LA)
   Plot: (1µm ≤ d ≤ 100µm)

(% always normalized: Σ = 100%)

fractal dimension from 2D sections

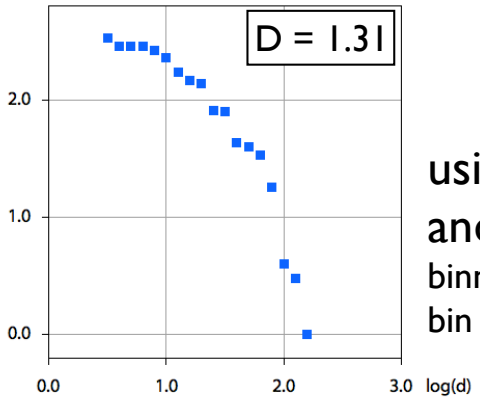




using linear diameters
and linear numbers
binning of d_{equ} (µm)
bin width Δd = constant

(linear data - plotted on linear x- and y- axes)

for sections
$D_{2d} \leq 2$

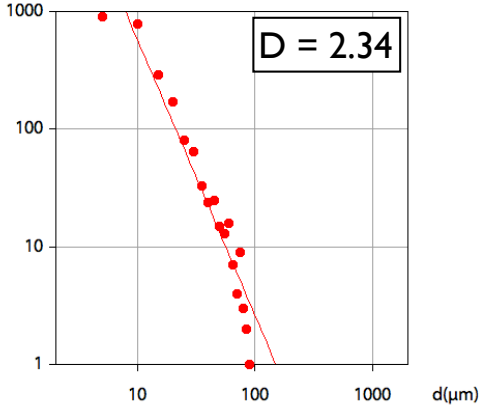


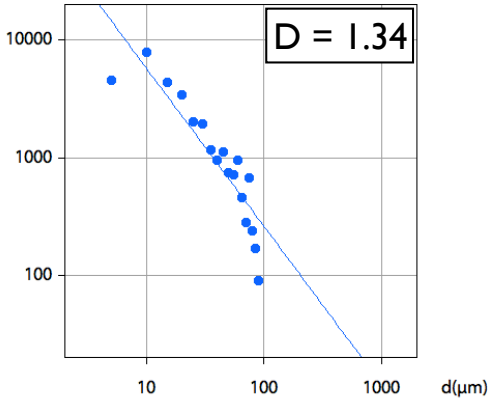
using log diameters
and log numbers
binning of log (d_{equ})
bin width $\Delta \log(d)$ = constant

(log data - plotted on linear x- and y- axes)

power law fit on linear data

n vs. $d \rightarrow D_{3d}$ $\quad (n \cdot d)$ vs. $d \rightarrow D_{2d}$

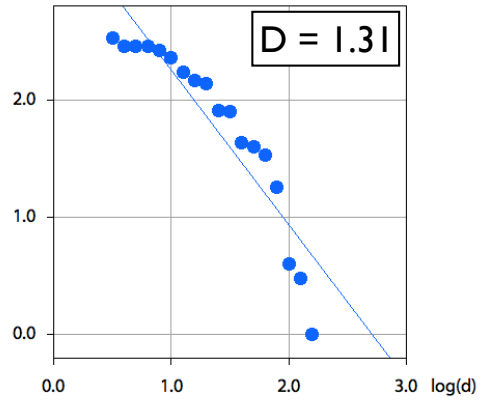




(linear data - plotted on logarithmic x- and y- axes)

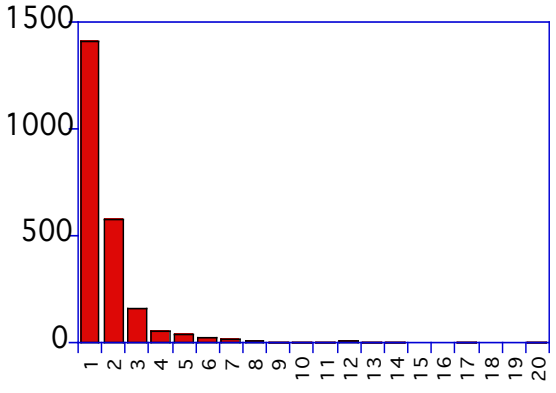
linear law fit on log data

$\log(n)$ vs. $\log(d) \rightarrow D_{2d}$

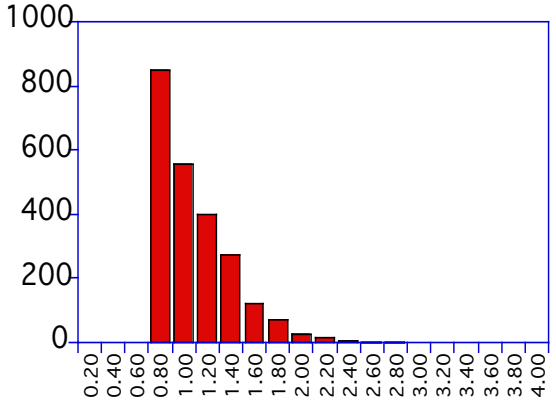


(log data - plotted on linear x- and y- axes)

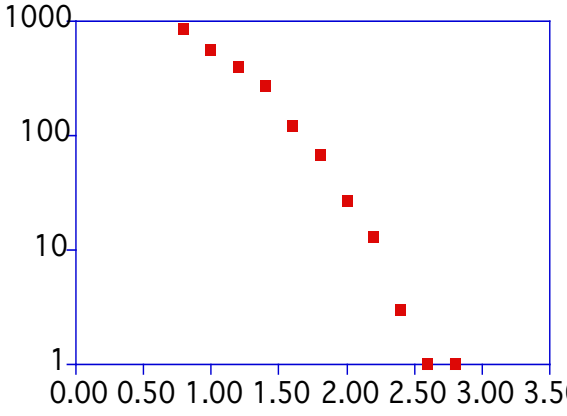
tricks of the trade



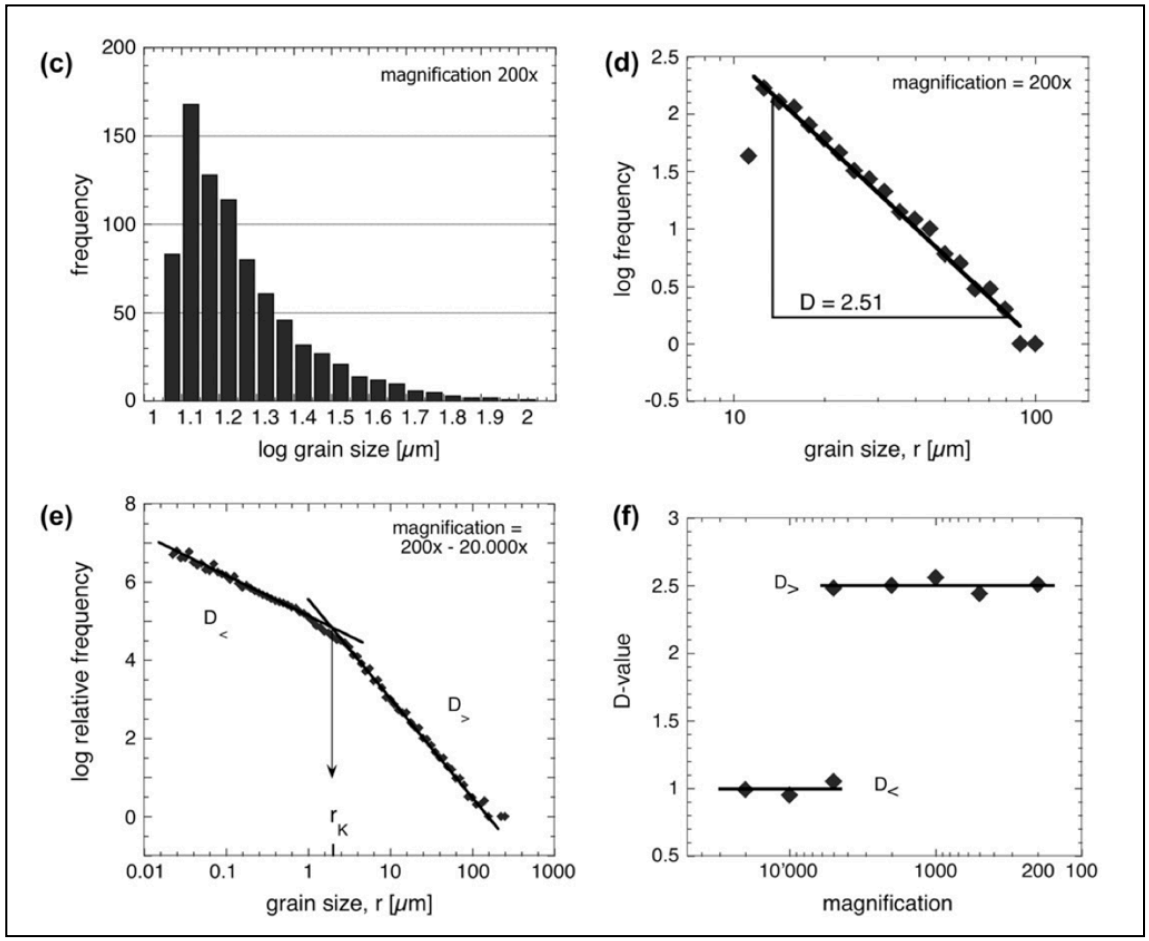
linear bins
linear counts



log bins
linear counts



log bins
log counts



Keulen, N., Heilbronner, R., Stünitz, H. and Boullier, A.-M. (2007). Grain size distributions of fault rocks: a comparison between experimentally and naturally deformed granitoids, Journal of Structural Geology, 29, 1282-1300, doi:10.1016/j.jsg.2007.04.003.

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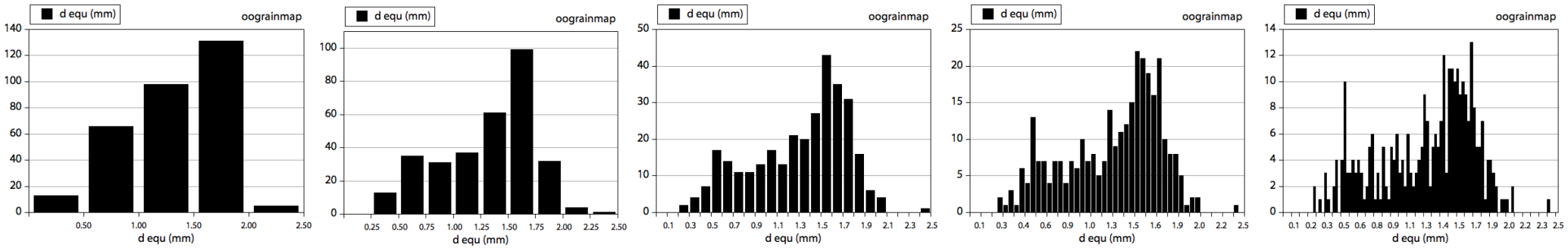
dos & donts

available / recommended programs

Software	what it does	where to get it
ImageJ / Fiji	Image analysis	https://fiji.sc/
Image SXM	Image analysis	https://www.liverpool.ac.uk/~sdb/ImageSXM/
stripstar (Fortran)	2D-3D grainsize analysis	https://micro.earth.unibas.ch → Software
Jazy_stripper	2D-3D grainsize analysis	https://github.com/kilir/Jazy_macros
PolyLX (python)	Microstructures analysis	https://github.com/ondrolexa/polylx
grain size toolbox (python)	Grain size analyis	https://marcoalopez.github.io/GrainSizeTools/
Matlab	Image processing toolbox	https://mathworks.com

resolution and significance

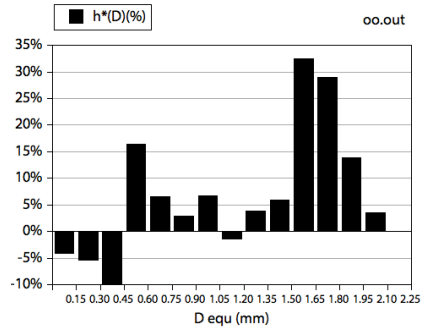


	5 bins	10 bins	20 bins	50 bins	100 bins
bin width (mm)	0.5	0.25	0.125	0.05	0.025
counts / bin	63	31	16	6	3

		as measured	as published
	n	313	
2D	mean (mm)	1.3075	1.31
2D	std.dev. (mm)	0.43268	± 0.43
3D	mode*) (mm)	1.7034	1.7
3D	std.dev. (mm)	0.11322	± 0.11

*) = mean of Gauss fit

use antispheres to estimate error



meaningful means

do	adapt bin size to number of measured sections
	should have approx. 20 counts per bin
do	calculate 3D grain size distributions
	= less biased than 2D distributions
do	use volume weighted histograms
	= phsically most meaningful
do	use the mode (or modes if bimodal)
	= dominant grain size(s)
don't	do not use non-linear bins !
	do not use log bins !
don't	do not use the mean, median, or rms
	= strongly dependent on shape of distribution
don't	do not use the 'correction factor' to estimate the 3D mean
	= useless

delicate fractal dimension

attention

when comparing data from particle analyzer (3D) to data obtained from thin section (2D)

warning

D_{2d} cannot be > 2.00
D_{3d} cannot be > 3.00

not every powerlaw fit is a fractal...

end
grainsize